

Hierarchical Eye-Tracking Data Analytics for Human Fatigue Detection at a Traffic Control Center

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Abstract—Eye-tracking-based human fatigue detection at traffic control centers suffers from an unavoidable problem of low-quality eye-tracking data caused by noisy and missing gaze points. In this study, the authors conducted pioneering work by investigating the effects of data quality on eye-tracking-based fatigue indicators and by proposing a hierarchical-based interpolation approach to extract the eye-tracking-based fatigue indicators from low-quality eye-tracking data. This approach adaptively classified the missing gaze points and hierarchically interpolated them based on the temporal-spatial characteristics of the gaze points. In addition, the definitions of applicable fixations and saccades for human fatigue detection were proposed. Two experiments were conducted to verify the effectiveness and efficiency of the method in extracting eye-tracking-based fatigue indicators and detecting human fatigue. The results indicated that most eye-tracking parameters were significantly affected by the quality of the eye-tracking data. In addition, the proposed approach could achieve much better performance than the classic velocity threshold identification algorithm (I-VT) and a state-of-the-art method (U'n'Eye) in parsing low-quality eye-tracking data. Specifically, the proposed method attained relatively stable eye-tracking-based fatigue indicators and reported the highest accuracy in human fatigue detection. These results are expected to facilitate the application of eye movement-based human fatigue detection in practice.

Index Terms—traffic management, eye movement, human fatigue, hierarchical-based interpolation

I. INTRODUCTION

Traffic control centers (TCC) maintain traffic flow and safety and decrease its environmental and economic impacts. Traffic control is implemented for all modes of transport, including vessel traffic service, air traffic management, freeway traffic management and rail traffic control [1-3]. Traffic control operators (TCOs) in TCCs face a high possibility of human fatigue [4], which can create disastrous outcomes for public safety [5, 6]. For example, in air traffic management, 13% of operational errors are directly caused by human fatigue [7].

Generally, the risk of human fatigue can be reduced by providing real-time fatigue monitoring of TCOs and alerting them when fatigue is detected. Previous studies have shown that eye movement patterns are effective fatigue indicators [8-10], and they can be tracked in an unconstrained and natural way. Moreover, eye movement patterns reveal patterns of human-system interaction, which is a desirable indicator for cognitive states [11].

Hence, many pilot studies have been conducted to detect human fatigue using eye-tracking data.

Although existing methods can efficiently analyze eye-tracking data, applying them in TCCs remains a challenge [12]. Due to the natural movements of TCOs when working, it is difficult to collect high-quality eye-tracking data. In addition, the illumination and reflection of visual displays can decrease the performance of eye trackers because of excess collected noise [13, 14]. It has been found that the quality of eye-tracking data has a severe effect on detecting fixations and saccades, which is normally the first step of eye-tracking-based human fatigue detection [15]. Specifically, the quality of the eye-tracking data will affect the number and duration of fixations, areas of interest and reaction latencies, and incomplete eye-tracking data can even lead to artificial saccades and fixations [13]. However, existing methods mainly use techniques that treat the missing gaze points as the end of a fixation or saccade [16], delete the uncertain fixations and saccades [13], replace the missing gaze points with average data [14, 17], define the fixations based on visual similarity [18], or even manually classify the fixations [19]. These techniques pay insufficient attention to the spatial and temporal characteristics of the gaze points, which are the significant features of eye-tracking data [20]. As a result, they are normally incapable of extracting accurate fixations and saccades from eye-tracking data [13].

Considering the above problems, this paper extends the concept of hierarchical interpolation to parse the eye-tracking data at TCCs. Hierarchical interpolation refers to decomposing the data into several levels or segments and then interpolating each segment separately [21]. Following this concept, this paper hierarchically segments the eye-tracking data based on the spatial and temporal characteristics of the gaze points. The following challenges should be addressed when adopting this approach: (1) Diverse data quality attributes will induce complex effects on the fatigue indicators. In addition, the quality of the eye-tracking data is affected by various factors (e.g., missing gaze points and noisy gaze points), leading to varied results on saccades and fixations. It is important and challenging to interpolate eye-tracking data by considering every aspect of data quality. (2) The dynamic spatial and temporal characteristics of the gaze points pose great challenges for interpolation. There are strong and dynamic correlations among consecutive gaze points. For example, the gaze points of fixation are

consecutive and located in a specified area [16]. Although these correlations provide guidance and opportunities to interpolate the data effectively and accurately, their highly dynamic properties are difficult to capture. (3) Eye-tracking-based fatigue indicators are sensitive to the techniques utilized to extract saccades and fixations [22].

Considering these challenges, this work proposes a method for parsing low-quality eye-tracking data and explores the following critical research problems: First, what are the eye-tracking-based fatigue indicators? Second, how does data quality affect these indicators? Furthermore, how can we hierarchically classify the missing gaze points considering the spatial and temporal characteristics?

To address these problems, we identified eye-tracking-based fatigue indicators via an extensive literature review, investigated the effects of data quality on these indicators and proposed a hierarchical-based interpolation method. The algorithm is extended based on a velocity threshold identification algorithm (I-VT) but does not use simple interpolation. In addition, definitions of applicable fixations and saccades for human fatigue detection are proposed. Two experiments showed that the innovative method can efficiently solve the problem of low-quality eye-tracking data. In other words, the proposed method can accurately extract the eye-tracking-based fatigue indicators from the eye-tracking data.

The literature on eye-tracking-based human fatigue detection and hierarchical interpolation is reviewed in Section 2. Section 3 provides a quantitative analysis of low-quality eye-tracking data for TCCs. The hierarchical eye-tracking data analytics for human fatigue detection at TCCs and the extraction of applicable fixations and saccades are presented in Section 4 and Section 5, respectively. Section 6 presents the experiments that were conducted to evaluate the performance of the proposed method. Section 7 concludes the paper and highlights future works.

II. LITERATURE REVIEW

This section provides a theoretical background from two aspects: eye-tracking-based human fatigue detection and applications of hierarchical interpolation.

A. Eye-tracking-based Human Fatigue Detection

Eye movements have been widely recognized as promising objective signals of cognitive fatigue [23]. Extensive studies have investigated the possibility of monitoring mental fatigue via eye-tracking data [12]. Basically, researchers first identify the saccades and fixations from the eye-tracking data and then generate the parameters from them [15], which are called eye-tracking-based fatigue indicators in this study. These indicators can be extracted from the basic saccades, such as saccade count [24, 25], saccade peak velocity [26], amplitude and saccade duration [27]. Saccade amplitude is the spatial length of the saccade. In addition, fixations have been found to be important eye-tracking-based fatigue indicators, such as fixation stability, fixation count, and

fixation duration [28]. Fixation stability is the mean variation of gaze positions.

Most of the existing algorithms for identifying saccades and fixations can be classified into one of two types: dispersion-based threshold algorithms and velocity threshold algorithms [14-16, 29, 30]. Dispersion-based threshold algorithms mark a set of consecutive points within a particular dispersion as a fixation [31]. For example, the I-DT, the most common dispersion-based algorithm, first determines a temporal window and then expands the window until the dispersion of the gaze points exceeds the threshold. Its dispersion is calculated based on the largest horizontal and vertical distances of the gaze points [22]. In other dispersion-based threshold algorithms, the dispersion can be measured as the distance between any two gaze points of the fixation [16], the distances between the points and the center of the fixation, and even the maximal horizontal distance plus the maximal vertical distance [29]. In addition to 2D fixation detection, Weber et al. [15] proposed a dispersion-based algorithm with an ellipsoidal bounding volume that estimates 3D fixations.

Velocity threshold algorithms separate the saccades and fixations based on velocity [16]. Normally, the velocities of fixation points are much lower than the velocities of saccadic points. Hence, the velocity of each gaze point is compared with a chosen threshold value. If its velocity is over the threshold, the gaze point is defined as a saccadic point. Otherwise, the gaze point is considered a fixation point. Although the velocity threshold algorithm is simple and intuitive, it is very sensitive to noise [22]. Liu et al. [14] proposed an integrated algorithm that combined the dispersion-based threshold algorithm and the velocity threshold algorithm to identify fixations in eye-tracking experiments.

In addition to dispersion-based threshold algorithms and velocity threshold algorithms, Steil et al. [18] proposed a method for fixation detection based on the visual similarity of gaze targets. However, this method heavily depends on the visual objects and cannot be used in monotonous target tracking tasks. In 2018, Bellet et al. [32] proposed a state-of-the-art method (U'n'Eye) based on a deep neural network to identify events and achieve human-level accuracy.

Considering the significant importance of the temporal-spatial-static characteristics from eye-movement trajectory data, Mukhopadhyay and Nandi [20] proposed a comprehensive framework called LPiTrack for eye-movement pattern discovery. Their study set a new direction for identifying eye-movement patterns, although they did not focus on fixation and saccade detection.

In sum, all the mentioned methods pay insufficient attention to the quality of raw eye tracking data. Thus, their results for the saccade and fixation parameters are sensitive to noisy and missing gaze points. Moreover, they adopt nonadaptive interpolation without regarding the characteristics of the gaze points. To overcome this problem, in this study, a hierarchical method is proposed to accurately extract the eye-tracking-based fatigue indicators from the eye-tracking data.

B. Hierarchical-based Data Analysis

Hierarchical interpolation refers to decomposing data into several levels or segments and then interpolating these segments separately [33, 34]. This concept has been successfully used in many fields, including electronic systems and hyperspectral image compression [34, 35].

As the characteristics of saccades and fixations are significantly different [28], hierarchically parsing them would be applicable. In general, the gaze points of fixation are consecutive and located in a specified area [18]. The gaze points of a saccade follow a Gaussian distribution [28]. Hence, researchers have tried to predict the trajectories for saccades and fixations separately. Feng et al. [36] modeled fixations with stable center points plus a zero-mean Gaussian random variable. They modeled saccades with linear regression plus a zero-mean Gaussian random variable [36]. Wass et al. [31] proposed a fixation detection algorithm that combines a number of criteria for separately detecting a genuine fixation. Following this trend, the authors of this paper propose hierarchically interpolating the fixations and saccades.

III. PROBLEM ANALYSIS

Eye-tracking data refer to a set of gaze points with timestamps, gaze velocities, and gaze positions. This can be an alternative measurement of mental fatigue at TCCs. Nevertheless, the eye-tracking data of TCOs suffer from missing and noisy gaze points. Missing gaze points refer to the data points with only timestamps and no gaze velocities or positions. Noisy gaze points are data points with abnormal gaze velocities or positions. In the following, a method for quantitatively measuring eye-tracking data quality is described first. Then, the effects of data quality on the eye-tracking fatigue indicators are investigated.

A. Definition of Eye-tracking Data Quality

Both noisy and missing gaze points will result in low-quality eye-tracking data, which significantly affects the values of the eye-tracking-based fatigue indicators. Quantitatively assessing the quality of eye-tracking data would provide a foundation for eye-tracking data analytics. The definition of eye-tracking data quality is thus introduced. Table 1 introduces the parameters and indexes that are used in this study.

A series of eye-tracking data (E) includes a set of timestamps, gaze positions, gaze velocities, and validity codes. In general, a validity code ranges from 0 to 4. A code of '0' indicates that the gaze point is certain and labeled as usable gaze data. Codes of '1', '2' and '3' indicate that the gaze point is not certain. A code of '4' indicates that no eye was identified, and the gaze point is missed. Following the definition of Tobii Guidance [37], the value derived by dividing the number of usable gaze data points by the number of attempts is defined as the eye-tracking data quality.

The quality of the eye-tracking data can be represented by $\{q, q_m, q_n\}$, where q represents the eye-tracking data

quality, q_m refers to the proportion of missing gaze points and q_n indicates the proportion of noisy gaze points.

Definition 1: Given the eye-tracking data E and the quality vector, the quality is defined as follows:

TABLE I
NOMENCLATURE

n	Number of gaze points ($n=1\dots,N$)
i	Number of gaze points that belong to gaps ($i=0,1\dots,n$).
j	Number of gaze points ($j=0,1,\dots,J, J<N-n-1$)
f	The sampling rate of the eye tracker.
E	A series of eye-tracking data.
q	The eye-tracking data quality.
q_m	The proportion of missing gaze points.
q_n	The proportion of noisy gaze points.
D_i	The direction of missing gaze point i .
GS_i	Refers to the gaze step between gaze point i and gaze point $i+1$.
GS_{max}	The maximum gaze step of the saccade.
T_{max}	The timestamp when the gaze step achieves the peak value.
St	The time width at half the maximum gaze step.
sg	Saccade gaze points.
fg	Fixation gaze points.
l_n	The label of gaze point n ; the value can be sg or fg .
U	The set of gaze points that cannot be interpolated.
x	The x coordinates of gaze points.
y	The y coordinates of gaze points.
e_n	The gaze point n .
t_n	Time stamp of gaze point n .
ga	The gap of eye-tracking data.
FD	Fixation duration.
FC	Fixation counts.
SD	Saccade duration.
SC	Saccade counts.
SPV	Saccade peak velocity.
SA	Saccade amplitude.

Class 1: if both the left and right eye-tracking data meet the requirements of $q_m + q_n < 5\%$ and $q > 95\%$, E are defined as high-quality eye-tracking data.

Class 2: if both the left and right eye-tracking data meet the requirements of $5\% < q_m + q_n < 10\%$ and $q > 95\%$, E are defined as acceptable eye tracking data.

Class 3: if both the left and right eye-tracking data meet the requirements of $20\% > q_m > 10\%$, $5\% > q_n$ and $q > 80\%$, E are defined as flicker eye-tracking data with many missing gaze points.

Class 4: if both the left and right eye-tracking data meet the requirements of $20\% > q_n > 10\%$, $5\% > q_m$ and $q > 80\%$, E are defined as flicker eye-tracking data with vague noisy data.

The definition of eye-tracking data quality is proposed to quantitatively evaluate the eye-tracking data and provide a reference for investigating the effects of eye-tracking defects. Instead of using a single quality value, the quality vector $\{q, q_m, q_n\}$ is introduced to evaluate the eye-tracking data quality.

B. Effect Analysis of Eye-tracking Data Quality

The eye-tracking data from 20 students with normal or corrected to normal visual function were collected. The average age of these students was 21 years old, and their eye movements were recorded when they were monitoring vessel traffic conditions and performing a target tracking task, which are similar to TCC operations. The task lasted for approximately two hours, which was sufficient to

record their fatigue status [40]. In addition, to obtain the ground truth of human fatigue, all the participants were required to report their objective fatigue levels using the Samn-Perelli scale at the beginning and end of the experiment.

The experiment was approved by an institutional review board with the reference number IRB-2018-04-007. The quality distribution of the eye-tracking data was analyzed and showed a serious data quality problem.

velocity of gaze points and misidentified saccades. The number of saccades increased significantly with a decrease in data quality ($p < 0.001$). The saccade amplitude in Class 4 was much higher than in other classes ($p < 0.05$).

The results were obtained by parsing the eye-tracking data with the standard velocity-based method. Hence, the peak velocity of fixation and mean velocity seemed to be stable across the four different data quality groups. Comparing the results for Class 3 and Class 4, fewer

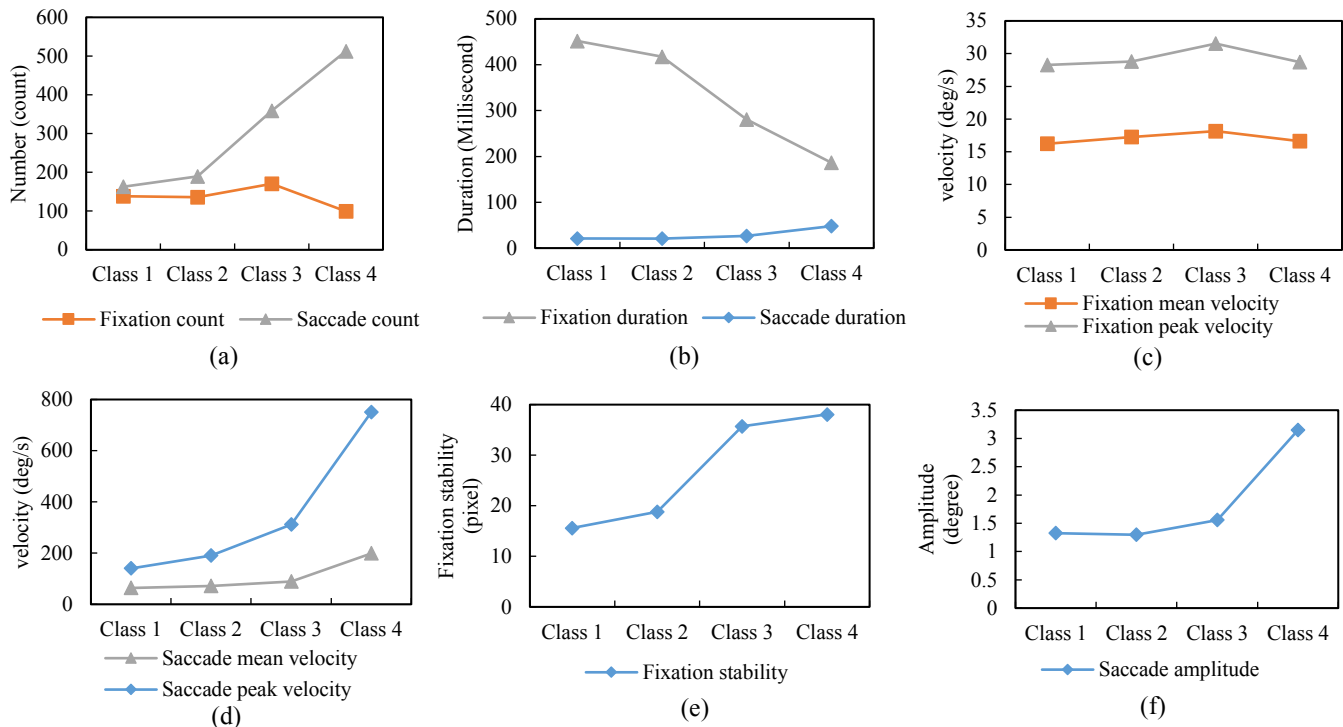


Fig. 1. Eye-tracking-based fatigue indicators obtained from four different data quality groups. (a) Fixation count and saccade count. (b) Fixation duration and saccade duration. (c) Fixation mean velocity and fixation peak velocity. (d) Saccade mean velocity and saccade peak velocity. (e) Fixation stability. (f) Saccade amplitude.

Many eye-tracking datasets have quality values lower than 80%. Moreover, the quality of the left and right eye data was different. Hence, it is necessary to evaluate the data quality of the left and right eyes separately.

Multivariate analysis of variance (MANOVA) was conducted to test the effects of data quality on eye-tracking-based fatigue indicators. For each subject, four parts of eye-tracking data belonging to different classifications were randomly selected. Some subjects may only provide high-class or low-class data. Their data were not selected. As a result, 48 parts of eye-tracking data from 12 subjects were selected.

Fig. 1 shows that there was no significant difference between Class 2 and Class 1 for almost all indicators except fixation duration and saccade peak velocity. Class 3 had many more fixations and saccades than Class 1 and Class 2 ($p < 0.05$). It is surprising that Class 4 had fewer fixations than other classes, and this result may be caused by the increased missing gaze points and noisy gaze points in Class 4. The missing gaze points lead to shorter and fewer fixations. The fixation duration and saccade duration decreased greatly with the decrease in data quality ($p < 0.001$). The noisy gaze points lead to a high

fixations and more saccades were found in Class 4. These results were caused by the high velocities induced by the noisy gaze points. The authors analyzed the raw data and found that almost all the gaze data labeled '1' to '3' had high velocities. The noisy gaze points with high velocity were regarded as saccadic data. Thus, there were more saccades and fewer fixations in Class 4. The pupil diameters were not significantly affected by the missing gaze points in this study. The missing gaze points did not affect the collected data of the pupil diameters, as the eye trackers collected enough pupil diameter data per second. Nevertheless, the noisy gaze points led to large pupil diameters.

The analysis indicated that when analyzing the eye-tracking data with the standard velocity-based method, the values of the eye-tracking-based fatigue indicators heavily depend on the data quality. The missing gaze points and noisy gaze points have different effects on these indicators. Hence, it can be concluded that noisy and missing gaze points are two dimensions of data quality and should be separately addressed. According to the results in Section 3.2, the traditional methods of parsing the eye-

tracking data will suffer from the problem of data quality. Hence, a novel method was proposed.

IV. HIERARCHICAL INTERPOLATION-BASED DATA ANALYSIS

An innovative method, named hierarchical-based eye-tracking data analytics (HEA), is proposed to overcome these challenges. Fig. 2 illustrates the framework of the proposed method. The raw data include timestamps, gaze positions, and gaze velocities. The problem of noisy gaze points is initially transformed into a problem of missing gaze points. The gaze points are classified into three types—fixation gaze points, saccade gaze points, and missing gaze points—by using existing methods. Then, the types of gaps are defined based on their neighboring gaze points. Hierarchical interpolation is conducted considering the types of gaps. The theories and detailed processes are described as follows.

A. Replacement of the Noisy Gaze Points

In general, noise in the eye-tracking data includes disparities and a great amount of variance caused by light reflection or other environmental factors.

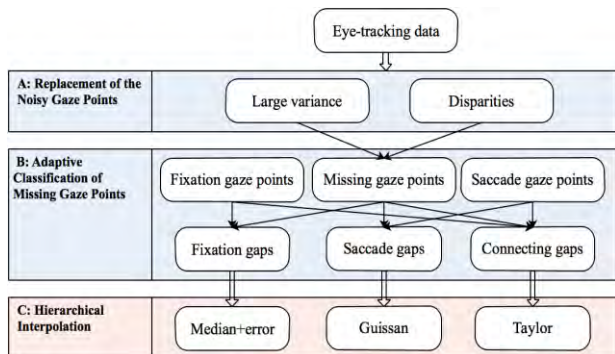


Fig. 2. The framework of hierarchical eye-tracking data analytics.

Specifically, given the eye-tracking data, the recorded velocity may show great variance due to noise. Hence, the median noise reduction algorithm is initially used to smooth the raw eye-tracking data. The velocity value of the gaze point is replaced by the median value of the neighborhood, which is called the mask.

The gaze points with binocular disparities can be identified by comparing the positions of the left and right eyes. If the distance between the left and right gaze points is larger than the precision of the eye tracker, the gaze points are treated as unreliable and replaced with gaps. A gaze sample whose peak velocity is faster than $600^\circ/\text{second}$ should be marked as unreliable and replaced with a gap, as the highest saccade peak velocity is not above $600^\circ/\text{second}$ [38]. After deleting all the noisy gaze points that meet the criteria mentioned above, the remaining gaze points from both eyes are merged into one dataset. Following the I-VT method, the average position data from the left and right eyes are calculated and saved for processing.

B. Adaptive Classification of Missing Gaze Points

The gaps caused by the missing gaze points may appear in fixations and saccades. Since the gaze sample

distributions of fixation and saccades are significantly different [28], the gaps may require different interpolation methods. Different from the traditional methods, the authors propose adaptively classifying the gaps into several types and then interpolating them separately. The details of the HEA method are shown in Fig. 2 and described as follows:

1) Gaze point classification

The gaze points are first classified into fixations and saccades. Then, the gaps are classified according to the types of their following and preceding gaze points.

The gaze points can be classified into fixations and saccades according to velocity or by visual inspections. The ‘velocity threshold’, which is utilized to distinguish between fixations and saccades, was set as $30^\circ/\text{s}^{-1}$, which is lower than the threshold used by some researchers [31]. There are two reasons for the low ‘velocity threshold’ utilized in this study. First, this ‘velocity threshold’ is sufficient for detecting fixations [37]. In addition, it can reduce the possibility of missing small saccades and maintain the stability of fixation. The stability of fixation is indicated by the variance of the fixation gaze points. Increasing the ‘velocity threshold’ will result in merging small saccades with fixations. The variance of saccades is significantly larger than that of fixations. Hence, the combination of small saccades with fixation will result in a large variance. The velocity of each gaze sample is checked against the ‘velocity threshold’. All the gaze samples whose velocities are under the ‘velocity threshold’ are marked as fixations. If the velocity of a gaze point exceeds the ‘velocity threshold’, the velocities of the neighboring gaze points are checked. If more than three gaze points have velocities higher than the ‘velocity threshold’, these gaze points are marked as saccades. Otherwise, the gaze points are marked as fixations.

Definition 2: Given a gaze point e_n , its label (l) is defined as follows:

Fixation gaze point (fg): e_n is a fixation gaze point if $v_n < 30^\circ/\text{second}$ or $\{v_{n-1}$ or $v_{n+1}\} < 30^\circ/\text{second}$, or defined as *fg* by experts.

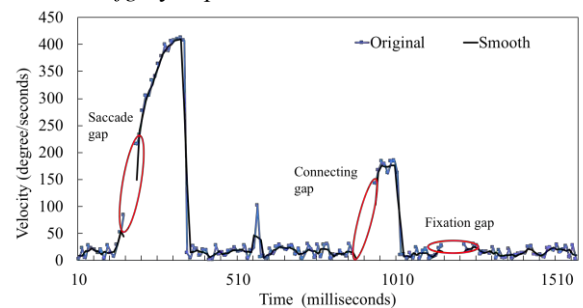


Fig. 3. Illustration of a saccade gap, connecting gap, and fixation gap.

Saccade gaze point (sg): e_n is a saccade gaze point if $\{v_{n-1}, v_n, v_{n+1}\} > 30^\circ/\text{second}$, or defined as *sg* by experts. *Missing gaze point:* e_n is a missing gaze point if $v_n \in \emptyset$.

2) Gap classification

A gap indicates a group of consecutive gaze points whose velocity is empty. The gaps are classified into four

types: fixation, saccade, connecting, and unrecoverable gaps.

An unrecoverable gap is identified using the ‘max interpolation length’, which means the maximum length of a gap that should be interpolated. A gap whose maximum length is shorter than the ‘max interpolation length’ is defined as a recoverable gap. Only the gaps caused by tracking problems, such as loss contact and reflection, can be interpolated. The gaps caused by blinking or looking away from the screen should be treated as ‘legitimate’ gaps. It is impossible and unnecessary to interpolate the ‘legitimate’ gaps, as no valid gaze points are available. The duration of a blink is approximately 100-400 ms [39]. Thus, the ‘max interpolation length’ should be set below 100 ms to avoid interpolating blink samples. In addition, the minimum duration of fixation is longer than 100 ms [24]. Hence, a ‘max gap length’ of 100 ms can avoid interpolating a complete fixation. The mean duration of a saccade ranges from 20 milliseconds to 200 milliseconds. Thus, a 100-ms gap may include a saccade. To avoid interpolating through a complete saccade, the distance of the gap is checked. If both the preceding and following gaze points of the gap are fixations, the distance between the two gaze points is divided by the number of missing gaze points in the gap. If the result is longer than 30° s^{-1} , the gap is regarded as an unrecoverable gap. In addition, the gaps whose lengths are longer than 100 ms are treated as unrecoverable gaps and will be deleted later. The gaps that fall between two consecutive fixation gaze samples are marked as fixation gaps. The recoverable gaps that fall between two consecutive saccadic gaze samples are marked as saccade gaps. The gaps between fixation and saccade are classified as connecting gaps. Fig. 3 illustrates these gaps, except for the unrecoverable gap.

Definition 3: For a series of consecutive gaze points $\{e_1, \dots, e_n\}$, a gap exists if $\{v_i, \dots, v_{i+j}\} \in \emptyset$. If $t_{i+j+1} - t_{i-1} < 100 \text{ ms}$:

Fixation gap: if $\{l_{i-1}, l_{i+j+1}\} = fg$ and the distance between the two gaze points is smaller than $30 \times (t_{i+j+1} - t_{i-1})$.

Saccade gap: if $\{l_{i-1}, l_{i+j+1}\} = sg$.

Connecting gap: if $l_{i-1} \neq l_{i+j+1}$.

Otherwise, it is an *unrecoverable gap*, U .

C. Hierarchical Interpolation

Different interpolating methods are applied to gaps in fixations and saccades, as their gaze sample distributions are significantly different [28]. The details for interpolating the gaps are described as follows:

1) Interpolating the fixation gaps

For idea fixation, it is assumed that the gaze points are located in the same position. Thus, many researchers have utilized the average data to fill the gaps. However, in reality, the gaze points vary in an area due to the hardware and microdrift or microsaccades. Hence, it is necessary to consider the deviation during interpolation. The positions of gaze points belonging to the fixation gaps can be obtained by summing the mean values of all the

neighboring fixation gaze points for the gap and the root mean square error (RMSE).

2) Interpolating the saccade gaps

The recoverable gaps that fall between two consecutive saccadic gaze samples are filled in with Gaussian fitting and prediction. The gaze steps of the saccades can be modeled with a Gaussian distribution [28]. The Gaussian distribution parameters for each saccade that requires gap-filling can be determined by implementing the expectation-maximization algorithm. With the Gaussian distribution parameters, the gaze steps between any two consecutive gaze points can be determined. As the gaze points have two dimensions, the direction of each gaze step should be determined. The gaze steps of the saccades are obtained, and then the Gaussian distribution parameters are estimated as follows:

$$GS_i = GS_{max} \times \exp[-(t_i - T_{max})^2 / St] \quad (1)$$

where GS_i refers to the gaze step between gaze point i and gaze point $i+1$. t_i is the timestamp of gaze point i . GS_{max} is the maximum gaze step of the saccade. T_{max} is the timestamp when the gaze step achieves the peak value. St refers to the time width at half the maximum gaze step. These parameters can be determined by implementing the expectation-maximization algorithm. The trajectory of the saccade is believed to be smooth. Hence, the moving direction (D) is estimated as the average direction of the valid gaze point before the gap and the first gaze point after the gap. The x- and y- coordinates of the gaze point can be determined as follows:

$$x_i = x_{i-1} + GS_i \times \cos(D) \quad (2)$$

$$y_i = y_{i-1} + GS_i \times \sin(D) \quad (3)$$

3) Interpolating the connecting gaps

Interpolating gaps that connect a fixation and a saccade is complex, as illustrated in Fig. 5. One of the challenges is determining the start or end point of fixation. To address this problem, the authors determine the start and end points of a saccade first. The Taylor series is applied to fill the gap of a saccade. The Gaussian distribution parameters and direction can be obtained by Equations (4) and (5), and then the x- and y-coordinates can be obtained by the results of Equations (2) and (3).

$$GS(i+1) = \sum_{n=0}^4 GS^{(n)}(i)/n! \quad (4)$$

$$D(i+1) = \sum_{n=0}^2 D^{(n)}(i)/n! \quad (5)$$

If the gaze step is smaller than the threshold, the gaze point is regarded as a fixation point. In this way, the end and start point of the fixation are confirmed. The same method described in Section 4.2.1 can be used to fill the gap.

V. FIXATION AND SACCADE IDENTIFICATION FOR HUMAN FATIGUE DETECTION

Fixations and saccades can be remotely tracked unobtrusively and continually during extended cognitive tasks using contactless eye trackers. In addition, the changes in fixations and saccades can be promptly captured to understand the interactions between the users and human interface devices. Hence, fixations and saccades are the most suitable biosignals for human

fatigue detection at a traffic control center. To this end, the identification of the fixations and saccades is necessary and critical, as this is the starting point for generating the eye-tracking-based fatigue indicators.

Given the eye-tracking data, saccades with a small spatial duration, fixations with a short temporal duration and incomplete fixations and saccades are eliminated first. Specifically, the saccades that are shorter than 2.5 degrees are eliminated. The normally used saccades range from 2.5 to 20 degrees, as an eye tracker with a low sampling rate cannot accurately collect small saccades. The fixations that are shorter than 100 milliseconds are eliminated, as a short fixation cannot provide enough information about the cognitive states. In addition, the fixations and saccades that are located before and after the unrecoverable gaps are eliminated. U refers to the set of gaze points that cannot be interpolated.

Definition 4: Given a series of consecutive gaze points $\{e_n, \dots, e_{n+j}\}$, a saccade can be identified if it satisfies (1) $\{l_n, \dots, l_{n+j}\} = sg$, (2) $\sum\{v_n, \dots, v_{n+j}\}/f > 2.5$, (3) $\{e_{n-1}, e_{n+j+1}\} \notin U$.

Definition 5: Given a series of consecutive gaze points $\{e_n, \dots, e_{n+j}\}$, a fixation can be identified if it satisfies (1) $\{l_n, \dots, l_{n+j}\} = fg$, (2) $t_{n+j} - t_n > 100 \text{ ms}$, (3) $\{e_{n-1}, e_{n+j+1}\} \notin U$.

Based on definitions 4 and 5, saccades and fixations can be identified first. Then, the saccade- and fixation-related indicators, including fixation duration, fixation count, fixation distribution, saccade peak velocity, saccade amplitude, and saccade count, are calculated. After that, machine learning methods such as decision trees and support vector machines can be used to discriminate human fatigue based on these fatigue indicators.

VI. EXPERIMENTS

The performance of the hierarchical eye-tracking data analytics (HEA) is evaluated from two aspects: 1) the efficiency in parsing simulated low-quality eye-tracking data and 2) the accuracy in detecting human fatigue. The HEA was compared with visual inspection and a deep neural network method (U'n'Eye [32]) in parsing eye-tracking data. The datasets and performance are explained in subsections A and B. For detecting human fatigue, the

HEA was compared with the I-VT and Tobii filters. Subsections C and D describe the experimental design and performance in detecting human fatigue.

A. Datasets of low-quality eye-tracking data

Three public datasets from Bellet et al. [32] were used to evaluate the performance of the HEA. Dataset 1 was collected from humans by an Eyelink 1000 video-based eye tracker with a 1000 Hz sampling rate. Dataset 2 was collected from monkeys with scleral search coils and a 1000 Hz sampling rate. Dataset 3 was collected from monkeys by an Eyelink 1000 video-based eye tracker with a 500 Hz sampling rate. The three datasets contain high-quality eye-tracking data without missing data. To generate low-quality simulation data, the three datasets were manipulated via random deletion and adding Gaussian noise. First, these raw data were replicated, and 10% of the data were randomly removed. Second, Gaussian noise (ranging from 0 to 17 cm) was added to 10% of the data. After these two steps, simulation data with a data quality of 80% were obtained. Meanwhile, the ground truth was set as the indicator extracted from the three datasets by visual inspection [32]. The values of fatigue indicators generated by the three methods are compared using a t-test with a significance level of 0.001.

B. Performance of Parsing Eye-tracking Data

Fig. 4 shows the fixation duration distribution of the three eye-tracking parsing methods. The HEA shows a similar fixation duration distribution with visual inspection. The HEA reported fewer short fixations and slightly more fixations longer than 300 ms. These results may be caused by merging small fixations and saccades. The fixation distribution reported by U'n'Eye was different from visual inspection and the HEA. It was found that for 300 ms fixations, U'n'Eye reported many fewer fixations than the other two methods but many more 500-ms fixations. This demonstrates that the HEA can detect fixations and saccades in parsing low-quality eye-tracking data. The statistical analysis of six fatigue indicators generated from three datasets is presented in Tables 2 to 4.

Table 2 shows the parameters of dataset 1. According to the t-test results, the HEA shows a significantly smaller RMSE than U'n'Eye for the six fatigue indicators, which

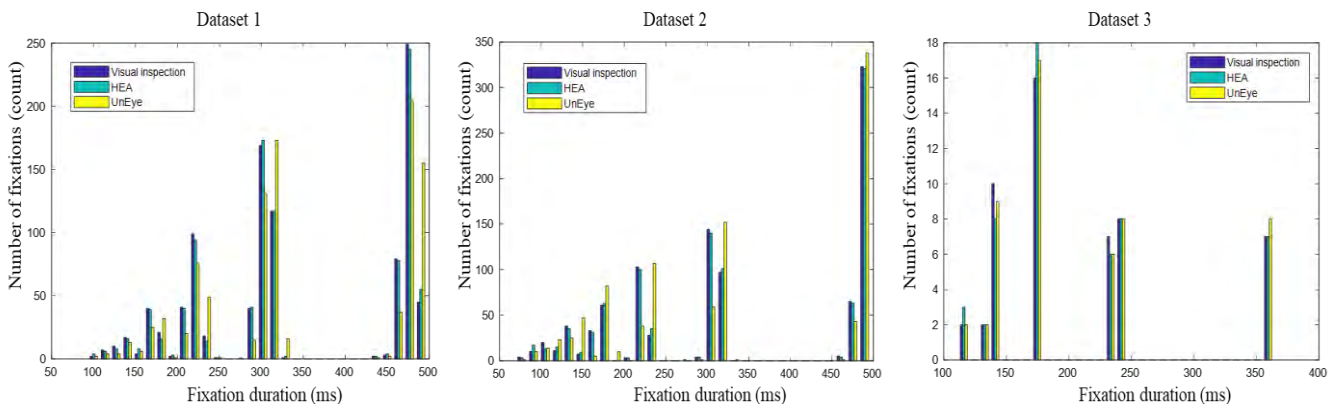


Fig. 4. Fixation duration distribution of three eye-tracking parsing methods.

means the HEA can provide higher accuracy in extracting fatigue indicators from low-quality eye-tracking data. Compared with the HEA and visual inspection, U'n'Eye obtained longer and fewer fixations from the simulation data. These results may be caused by the misclassification of missing gaze points. For example, U'n'Eye tends to classify the missing gaze points as fixation points that result in longer fixation. In addition, the HEA obtained fewer fixations than visual inspection. This may be caused by the last step of selecting the fixations and saccades, in which all the small fixations were eliminated by the HEA.

For saccade identification, the HEA provided longer and more saccades than U'n'Eye. This may be because 1) the HEA merged the small saccades into one saccade, which makes the saccade duration relatively longer than

TABLE II
DATASET 1 PARAMETERS GENERATED BY THREE METHODS

Method	FD	FC	SD	SC	SPV	SA
VI	352.9	2.9	42.9	1.9	128.1	16.5
HEA	357.6*	2.9	43.0	1.9	127.5	16.5
U'n'Eye	373.7*	2.8*	36.4*	1.8*	199.5	16.3
HEA_RMSE	42.3	0.3	4.5	0.3	1.8	0.2
U_RMSE	92.6	0.6	11.5	0.6	113.2	0.8
p	0.0	0.0	0.0	0.00	0.05	0.00

Note: VI: visual inspection, HEA: the proposed method, U'n'Eye: a deep neural network, HEA_RMSE: the root mean squared error between HEA and VI, U_RMSE: the root mean squared error between U'n'Eye and VI, FD: mean fixation duration, FC: fixation counts, SD: saccade duration, SC: saccade counts, SPV: saccade peak velocity, SA: saccade amplitude. p: the t-test results between HEA_RMSE and U_RMSE. *: the significant difference between the observed value and target value at level 0.001.

TABLE III
DATASET 2 PARAMETERS GENERATED BY THREE METHODS

Method	FD	FC	SD	SC	SPV	SA
VI	350.4	3.2	34.8	2.2	80.8	11.8
HEA	354.0	3.1	34.3*	2.2*	80.8	11.7
U'n'Eye	358.8*	3.3*	28.9*	2.3*	78.4*	10.1*
HEA_RMSE	22.9	0.2	2.8	0.2	0.1	0.1
U_RMSE	114.3	0.9	9.3	0.9	1.4	0.3
p	0.00	0.00	0.00	0.00	0.05	0.0

Note: VI: visual inspection, HEA: the proposed method, U'n'Eye: a deep neural network, HEA_RMSE: the root mean squared error between HEA and VI, U_RMSE: the root mean squared error between DNN and VI, FD: mean fixation duration, FC: fixation counts, SD: saccade duration, SC: saccade counts, SPV: saccade peak velocity, SA: saccade amplitude. p: the t-test results between HEA_RMSE and U_RMSE. *: the significant difference between the observed value and target value at level 0.001.

TABLE IV
DATASET 3 PARAMETERS GENERATED BY THREE METHODS

Method	FD	FC	SD	SC	SPV	SA
VI	217.3	3.7	11.1	2.6	128.9	5.0
HEA	216.5	3.7	10.9	2.6	129.4	4.9
U'n'Eye	220.3	3.6	11.1	2.6	128.7	5.0
HEA_RMSE	10.2	0.2	0.5	0.2	0.3	0.03
U_RMSE	22.2	0.3	0.9	0.3	0.6	0.03
p	0.18	0.18	0.30	0.18	0.32	0.88

Note: VI: visual inspection, HEA: the proposed method, U'n'Eye: deep neural network, HEA_RMSE: the root mean squared error between HEA and VI, U_RMSE: the root mean squared error between U'n'Eye and VI, FD: mean fixation duration, FC: fixation counts, SD: saccade duration, SC: saccade counts, SPV: saccade peak velocity, SA: saccade amplitude. p: the t-test results between HEA_RMSE and U_RMSE. *: the significant difference between the observed value and target value at level 0.001.

U'n'Eye, and 2) the HEA tends to interpolate missing gaze points and thus identifies more saccades than U'n'Eye.

For dataset 2, all six fatigue indicators generated by U'n'Eye were significantly different from the results generated by visual inspection. The results revealed the negative effects of missing gaze points on the efficiency of eye-tracking parsing methods. In addition, the HEA achieved better performance than U'n'Eye, by which no significant effects were found on fixation duration, fixation counts, and saccade peak velocity. Moreover, the RMSE of U'n'Eye was significantly larger than that of HEA.

The relationship between saccade peak velocity and saccade amplitude is frequently used to detect human fatigue [27]. Hence, we investigated the performance of the HEA in identifying saccade peak velocity and saccade amplitude. From datasets 1 and 2, the HEA could correctly identify saccade peak velocity and saccade amplitude. However, U'n'Eye could provide lower saccade peak velocity and shorter saccade amplitude due to the missing gaze points, as shown in Tables 2 and 3.

For dataset 3, no significant difference was observed in the six fatigue indicators and RMSE, as shown in Table 4. As dataset 3 only has 53 samples, while the other two datasets have 1000 samples, the small sample size should be the cause of no significant difference. Considering the results of datasets 1 to 3, the fixation counts should not be

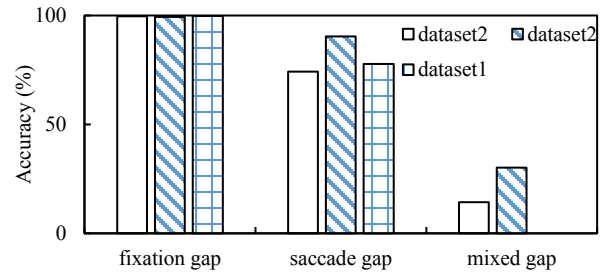


Fig. 5. Accuracy of gap classification.

used as fatigue indicators because they depend heavily on the parsing method and data quality. In contrast, the fixation duration could be a potential fatigue indicator when using the HEA.

Fig. 5 shows the patterns of correctly classified and interpolated gaps. Most of the fixation gaps can be accurately identified and filled in all three datasets with an accuracy above 90%. For saccade gaps, the classification accuracy is acceptable at 74.17%, 90.41%, and 77.78% for datasets 1 to 3, respectively. Nevertheless, connecting gaps were normally misclassified in dataset 1 and dataset 2, with accuracies of 14.3% and 30.1%, respectively. The sample size of dataset 3 is too small, and no connecting gap was generated. Hence, no accuracy results of the connecting gap were obtained for dataset 3. In conclusion, the HEA can deal with fixation gaps but does not perform well in interpolating connecting gaps.

C. Experimental Design of Eye-tracking-based Fatigue Detection

In this section, the performance of human fatigue detection using the eye-tracking fatigue indicators generated by three methods, the Tobii filter, the HEA, and

the standard I-VT, was investigated. The process of human fatigue detection is as follows. Tobii X3-120 sent the raw eye-tracking data to a ThinkPad X1 with MATLAB 2017, which was used to parse the raw eye-tracking data and detect human fatigue [12]. Specifically, the first and last five minutes of the eye-tracking data of 20 participants were parsed by the three methods and labeled with alert and fatigue, respectively. A support vector machine (SVM) was used to build the models and the associated eye-tracking-based fatigue indicators with fatigue labels. The accuracies of the SVM models were compared and tested by repeated measures analysis of variance (ANOVA).

D. The Performance of Human Fatigue Detection

Table 5 shows the performance of the SVM models using the eye-tracking fatigue indicators generated by the four methods. SVM is the most widely used machine learning method for eye-movement-based human fatigue detection [12, 40]. In previous studies, SVM was reported to be a suitable method for eye-tracking-based fatigue detection and achieved an accuracy of approximately 70% [41, 42].

Repeated measures ANOVA was conducted to test the within-in-subject effects. Significant effects were observed with $F_{(2,57)} = 21.35$ and $p = 0.00 < 0.05$. The HEA

TABLE V
PERFORMANCE OF HUMAN FATIGUE DETECTION

Method	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Tobii filter	79.6	6.63	76.7	82.5
I_VT	82.2	6.77	79.1	85.3
HEA	84.6	3.19	83.3	86.0

achieved the highest accuracy of 84.6% and the lowest standard error of 3.19. In conclusion, the HEA significantly improved the performance of using eye-tracking data to detect human fatigue.

E. Discussion

According to the above results, several implications can be concluded as follows.

First, the HEA can accurately elicit fatigue indicators, including saccade peak velocity, saccade amplitude, and fixation duration from low-quality eye-tracking data.

Second, compared with the standard I-VT method and U'n'Eye, the proposed method performs better in parsing low-quality eye-tracking data. It can reduce artificial events by merging the short gaze samples or eliminating the unreliable gaze samples.

Third, the HEA can retain the most valid data and obtain almost the same distributions of fixations and saccades from the simulation data and the original data. Compared with traditional methods, more valid data can be retained by the HEA.

Finally, the eye-tracking fatigue indicators generated by the HEA achieve the best performance in human fatigue detection.

Nevertheless, there are also some limitations in real applications. On the one hand, the proposed method can well interpolate the eye-tracking data with data quality

above 80%, and the performance decreases when the data quality is lower than 80%. On the other hand, although the proposed method provides effective eye-tracking-based indicators for human fatigue detection, it should be noted that simply using eye-tracking-based indicators cannot fully reveal all aspects of human fatigue states. Integrating these indicators with data for other indicators, such as working conditions, sleep time, brain dynamics, and heart rate, would greatly improve the performance of human fatigue detection [43].

VII. CONCLUSION

Researchers have paid limited attention to eye-tracking-based human fatigue detection for TCOs, whose eye-tracking data always suffer from the problem of missing and noisy gaze points. To solve this problem, this work proposed a hierarchical eye-tracking data analysis method. Two experiments were conducted to compare its performance with the I-VT and a novel method, U'n'Eye.

This study extends the research on human fatigue management for TCCs in several dimensions. First, this innovative approach is superior to existing methods for attaining eye-tracking fatigue indicators. This approach facilitates discriminating human fatigue using low-quality eye-tracking data and can be extended to generate the parameters for other fields, such as workload and attention evaluation. Second, this is pioneering work that investigates the negative effects of missing and noisy gaze points on eye-tracking-based fatigue indicators. This investigation provides references for discriminating human fatigue using eye movement. In addition, this research indicates the necessity for considering data quality in eye movement-based studies.

In the future, this work can be improved and extended from the following aspects. First, a study on accurately measuring microsaccade parameters using an eye tracker with a low-frequency sampling rate can be conducted. Moreover, the performance in detecting saccade gaps and connecting gaps can be improved. In addition, experiments with real traffic control operators at typical TCCs should be conducted to facilitate its adoption in practice.

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REFERENCES

- [1] A. Kouvelas, K. Aboudolas, E. B. Kosmatopoulos, and M. Papageorgiou, "Adaptive performance optimization for large-scale traffic control systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 12, no. 4, pp. 1434-1445, 2011.

- [2] O. D. Turnbull and A. G. Richards, "Human Control of Air Traffic Trajectory Optimizer," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 4, pp. 1091-1099, 2018.
- [3] G. Xu, F. Li, and C.-H. Chen, "Local AIS data analytics for efficient operation management in Vessel Traffic Service," in *Automation Science and Engineering (CASE), 2017 13th IEEE Conference on*, 2017, pp. 1668-1672: IEEE.
- [4] S. Charbonnier, R. N. Roy, S. Bonnet, and A. Campagne, "EEG index for control operators' mental fatigue monitoring using interactions between brain regions," *Expert Systems with Applications*, vol. 52, pp. 91-98, 2016.
- [5] Q. Ji, P. Lan, and C. Looney, "A probabilistic framework for modeling and real-time monitoring human fatigue," *IEEE Transactions on systems, man, and cybernetics-Part A: Systems and humans*, vol. 36, no. 5, pp. 862-875, 2006.
- [6] M. J. Akhtar and I. B. Utne, "Human fatigue's effect on the risk of maritime groundings—A Bayesian Network modeling approach," *Safety science*, vol. 62, pp. 427-440, 2014.
- [7] L.-p. Yuan, G.-f. Ma, and R.-s. Sun, "An Analysis of Fatigue and Its Characteristics: A Survey on Chinese Air Traffic Controller," in *International Conference on Engineering Psychology and Cognitive Ergonomics*, 2016, pp. 38-47: Springer.
- [8] R. Wierds, M. J. Janssen, and H. Kingma, "Measuring saccade peak velocity using a low-frequency sampling rate of 50 Hz," *IEEE Transactions on Biomedical Engineering*, vol. 55, no. 12, pp. 2840-2842, 2008.
- [9] R. Zheng, K. Nakano, H. Ishiko, K. Hagita, M. Kihira, and T. Yokozeki, "Eye-Gaze Tracking Analysis of Driver Behavior While Interacting With Navigation Systems in an Urban Area," *IEEE Trans. Human-Machine Systems*, vol. 46, no. 4, pp. 546-556, 2016.
- [10] Y. B. Eisma, C. D. Cabrall, and J. C. de Winter, "Visual Sampling Processes Revisited: Replicating and Extending Senders (1983) Using Modern Eye-Tracking Equipment," *IEEE Transactions on Human-Machine Systems*, no. 99, pp. 1-15, 2018.
- [11] A. Pimenta, D. Carneiro, J. Neves, and P. Novais, "A neural network to classify fatigue from human-computer interaction," *Neurocomputing*, vol. 172, pp. 413-426, 2016.
- [12] F. Li, C.-H. Lee, C.-H. Chen, and L. P. Khoo, "Hybrid data-driven vigilance model in traffic control center using eye-tracking data and context data," *Advanced Engineering Informatics*, vol. 42, p. 100940, 2019.
- [13] P. Blignaut and D. Wium, "Eye-tracking data quality as affected by ethnicity and experimental design," *Behavior research methods*, vol. 46, no. 1, pp. 67-80, 2014.
- [14] B. Liu, Q.-C. Zhao, Y.-Y. Ren, Q.-J. Wang, and X.-L. Zheng, "An elaborate algorithm for automatic processing of eye movement data and identifying fixations in eye-tracking experiments," *Advances in Mechanical Engineering*, vol. 10, no. 5, p. 1687814018773678, 2018.
- [15] S. Weber, R. S. Schubert, S. Vogt, B. M. Velichkovsky, and S. Pannasch, "Gaze3DFix: Detecting 3D fixations with an ellipsoidal bounding volume," *Behavior research methods*, vol. 50, no. 5, pp. 2004-2015, 2018.
- [16] P. Blignaut, "Fixation identification: The optimum threshold for a dispersion algorithm," *Attention, Perception, & Psychophysics*, vol. 71, no. 4, pp. 881-895, 2009.
- [17] S. V. Wass, L. Forssman, and J. Leppänen, "Robustness and precision: How data quality may influence key dependent variables in infant eye-tracker analyses," *Infancy*, vol. 19, no. 5, pp. 427-460, 2014.
- [18] J. Steil, M. X. Huang, and A. Bulling, "Fixation detection for head-mounted eye tracking based on visual similarity of gaze targets," in *Proceedings of the 2018 ACM Symposium on Eye Tracking Research & Applications*, 2018, p. 23: ACM.
- [19] I. T. Hooge, D. C. Niehorster, M. Nyström, R. Andersson, and R. S. Hessels, "Is human classification by experienced untrained observers a gold standard in fixation detection?," *Behavior research methods*, vol. 50, no. 5, pp. 1864-1881, 2018.
- [20] S. Mukhopadhyay and S. Nandi, "LPiTrack: Eye movement pattern recognition algorithm and application to biometric identification," *Machine Learning*, vol. 107, no. 2, pp. 313-331, 2018.
- [21] E. M. Reingold, "Eye tracking research and technology: Towards objective measurement of data quality," *Visual cognition*, vol. 22, no. 3-4, pp. 635-652, 2014.
- [22] M. Nyström and K. Holmqvist, "An adaptive algorithm for fixation, saccade, and glissade detection in eyetracking data," *Behavior research methods*, vol. 42, no. 1, pp. 188-204, 2010.
- [23] V. Renata, F. Li, C.-H. Lee, and C.-H. Chen, "Investigation on the Correlation between Eye Movement and Reaction Time under Mental Fatigue Influence," in *2018 International Conference on Cyberworlds (CW)*, 2018, pp. 207-213: IEEE.
- [24] R. Schleicher, N. Galley, S. Briest, and L. Galley, "Blinks and saccades as indicators of fatigue in sleepiness warnings: looking tired?," *Ergonomics*, vol. 51, no. 7, pp. 982-1010, 2008.
- [25] L. M. Bergasa, J. Nuevo, M. A. Sotelo, R. Barea, and M. E. Lopez, "Real-time system for monitoring driver vigilance," *IEEE Transactions on Intelligent Transportation Systems*, vol. 7, no. 1, pp. 63-77, 2006.
- [26] T. Morris and J. C. Miller, "Electrooculographic and performance indices of fatigue during simulated flight," *Biological psychology*, vol. 42, no. 3, pp. 343-360, 1996.
- [27] L. L. Di Stasi, R. Renner, A. Catena, J. J. Cañas, B. M. Velichkovsky, and S. Pannasch, "Towards a driver fatigue test based on the saccadic main sequence: A partial validation by subjective report data," *Transportation research part C: emerging technologies*, vol. 21, no. 1, pp. 122-133, 2012.
- [28] R. S. Bogartz and A. Staub, "Gaze step distributions reflect fixations and saccades: A comment on Stephen and Mirman (2010)," *Cognition*, vol. 123, no. 2, pp. 325-334, 2012.
- [29] F. Shic, B. Scassellati, and K. Chawarska, "The incomplete fixation measure," in *Proceedings of the 2008 symposium on Eye tracking research & applications*, 2008, pp. 111-114: ACM.
- [30] R. Engbert and R. Kliegl, "Microsaccades uncover the orientation of covert attention," *Vision research*, vol. 43, no. 9, pp. 1035-1045, 2003.
- [31] S. V. Wass, T. J. Smith, and M. H. Johnson, "Parsing eye-tracking data of variable quality to provide accurate fixation duration estimates in infants and adults," *Behavior Research Methods*, vol. 45, no. 1, pp. 229-250, 2013.
- [32] M. E. Bellet, J. Bellet, H. Nienborg, Z. M. Hafed, and P. Berens, "Human-level saccade detection performance using deep neural networks," *Journal of neurophysiology*, vol. 121, no. 2, pp. 646-661, 2018.
- [33] W. Yang and J. Feng, "2D shape morphing via automatic feature matching and hierarchical interpolation," *Computers & Graphics*, vol. 33, no. 3, pp. 414-423, 2009.
- [34] I. Ukhov, P. Eles, and Z. Peng, "Probabilistic analysis of electronic systems via adaptive hierarchical interpolation," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 36, no. 11, pp. 1883-1896, 2017.
- [35] M. Gashnikov and N. Glumov, "Hierarchical grid interpolation for hyperspectral image compression," *Computer Optics*, vol. 38, no. 1, pp. 87-93, 2014.
- [36] Y. Feng, G. Cheung, W.-t. Tan, and Y. Ji, "Hidden markov model for eye gaze prediction in networked video streaming," in *Multimedia and Expo (ICME), 2011 IEEE International Conference on*, 2011, pp. 1-6: IEEE.
- [37] A. Olsen, "The Tobii I-VT fixation filter," *Tobii Technology*, 2012.
- [38] D. Boghen, B. Troost, R. Daroff, L. Dell'Osso, and J. Birkett, "Velocity characteristics of normal human saccades," *Investigative Ophthalmology & Visual Science*, vol. 13, no. 8, pp. 619-623, 1974.
- [39] S. Benedetto, M. Pedrotti, L. Minin, T. Baccino, A. Re, and R. Montanari, "Driver workload and eye blink duration," *Transportation research part F: traffic psychology and behaviour*, vol. 14, no. 3, pp. 199-208, 2011.
- [40] Y. Yamada and M. Kobayashi, "Detecting mental fatigue from eye-tracking data gathered while watching video," in *Conference on Artificial Intelligence in Medicine in Europe*, 2017, pp. 295-304: Springer.
- [41] Y. Yamada and M. Kobayashi, "Detecting mental fatigue from eye-tracking data gathered while watching video: evaluation in younger and older adults," *Artificial intelligence in medicine*, vol. 91, pp. 39-48, 2018.

- [42] A. S. Zandi, A. Quddus, L. Prest, and F. J. Comeau, "Non-Intrusive Detection of Drowsy Driving Based on Eye Tracking Data," *Transportation Research Record*, p. 0361198119847985, 2019.
- [43] H. Wang, C. Wu, T. Li, Y. He, P. Chen, and A. Bezerianos, "Driving fatigue classification based on fusion entropy analysis combining EOG and EEG," *IEEE Access*, vol. 7, pp. 61975-61986, 2019.