Post-Quantum Cryptography

Dr. Ruben Niederhagen, February 8, 2016

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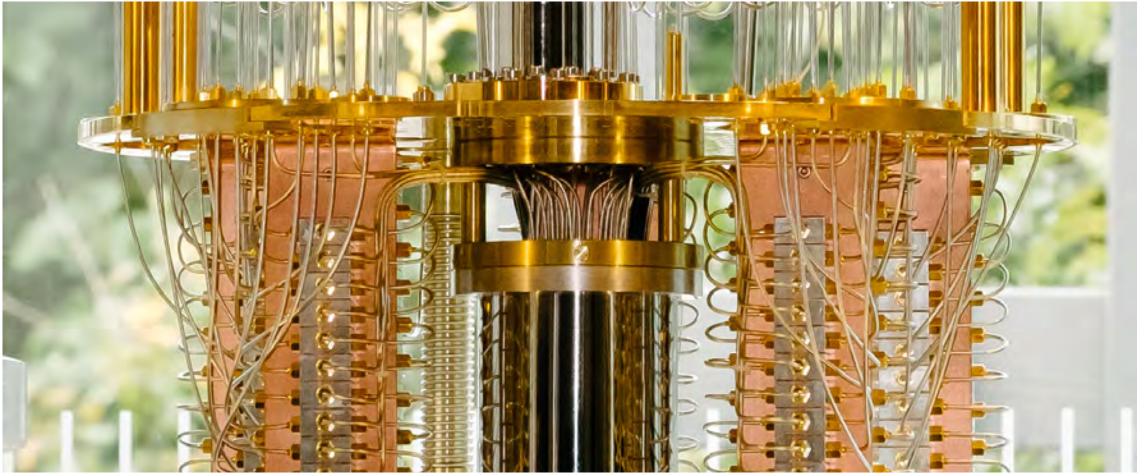

Introduction
Quantum Computers

Using quantum states for computation:

Introduced in 1985 by David Deutsch [3].

- Operate on *qubits*
- using *gates*
- that perform *reversible* operations
- exploiting *entanglement* and *superposition*.

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Theoretical (since $\approx$ 1900):

- qubit: $\mathbb{C}^2$
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Physical (since $\approx$ 1990s):

- qubit: photon, electron, atom, quantum dots...
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Quantum algorithms:

- Simon's algorithm, Deutsch–Jozsa algorithm, . . .
- Grover's algorithm: search in $\sqrt{n}$ time.
- Shor's algorithm: discrete logarithm and integer factorization in polynomial time (solve the abelian hidden subgroup problem).

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- Grover reduces a brute force attack on AES-128 from time $c \cdot 2^{128}$ to time $c' \cdot 2^{64}$; similar for hash-functions.
  $\Rightarrow$ **Use 256-bit primitives!**

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  $\Rightarrow$ **Use 256-bit primitives!**
- Shor **breaks all RSA, ECC, DHE, ECDHE, DSA, ECDSA, ..!**

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The Internet is broken, secure communication is broken; what now?

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Use quantum technologies to fight quantum technology!

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The cryptographer says:

Just base your crypto on math that quantum computers can't break.

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- does not work for mobile phones, sensor networks, cars, ...
- does not scale well, and
- is not really necessary if one does not insist in *physical principles* but is fine with **math and computational complexity**.

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Post-Quantum Cryptography

Main task of post-quantum cryptography [2]:

Find mathematically hard problems that

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- cannot be broken by classical computers,
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- provide a trapdoor for asymmetric crypto, and
- can be used efficiently in terms of
  - time,
  - memory, and
  - communication.

Introduction
Post-Quantum Cryptography

Current approaches are:

- code-based cryptography,
- multivariate cryptography,
- hash-based cryptography,
- lattice-based cryptography, and
- supersingular elliptic curve isogenies.

Code-based Cryptography

Error correction on a noisy channel:

Add redundant information to the message that allows to detect and correct bit-errors.

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Code-based Cryptography
Error-Correcting Codes

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Encoding: Multiply message vector with *generator matrix*.

Decoding: Use *decoding algorithm* of the code.

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Code-based Cryptography
McEliece Crypto System

- System Parameters: $n, t \in \mathbb{N}$, where $t \ll n$.
- Key Generation:

$$\begin{aligned} \mathsf{G} &: \ k \times n \text{ generator matrix of a code } \mathcal{G}, \\ \mathsf{S} &: \ k \times k \text{ random non-singular matrix}, \\ \mathsf{P} &: \ n \times n \text{ random permutation matrix}. \end{aligned}$$

  Compute $k \times n$ matrix $\mathsf{G}^{\mathsf{pub}} = \mathsf{SGP}$.

- Public Key: $(\mathsf{G}^{\mathsf{pub}}, t)$
- Private Key: $(\mathsf{S}, D_{\mathcal{G}}, \mathsf{P})$
  where $D_{\mathcal{G}}$ is an efficient decoding algorithm for $\mathcal{G}$.

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- Encryption: to encrypt message $\mathsf{m} \in \mathbb{F}_2^k$,
  randomly choose $\mathsf{e} \in \mathbb{F}_2^n$ of weight t; compute

$$\mathsf{c} = \mathsf{m}\mathsf{G}^{\mathsf{pub}} \oplus \mathsf{e}.$$

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- Decryption: compute

$$\mathsf{c}' = \mathsf{c}\mathsf{P}^{-1} = \mathsf{mSG} \oplus \mathsf{e}\mathsf{P}^{-1},$$

use $D_{\mathcal{G}}$ to decode c' to $\mathsf{m}' = \mathsf{mS}$,
compute

$$\mathsf{m} = \mathsf{m}'\mathsf{S}^{-1} = \mathsf{mSS}^{-1}.$$

McEliece problem:

Given a McEliece public key $(\mathsf{G}^{\mathsf{pub}}, t)$, $\mathsf{G}^{\mathsf{pub}} \in \{0,1\}^{k \times n}$ and a cipher text $\mathsf{c} \in \{0,1\}^n$, find a message $\mathsf{m} \in \{0,1\}^k$ with $w_{\mathsf{H}}(\mathsf{m}\mathsf{G}^{\mathsf{pub}} - \mathsf{c}) = t$.

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The hardness of this problem depends on the specific code.
McEliece proposes to use binary Goppa codes.

Code-based Cryptography
Niederreiter Crypto System

- System Parameters: $n, t \in \mathbb{N}$, where $t \ll n$.
- Key Generation:
  H : $(n-k) \times n$ parity check matrix of a code $\mathcal{G}$,
  P : $n \times n$ random permutation matrix.
  Compute
  S : $(n-k) \times (n-k)$ non-singular matrix, and
  $\mathrm{H}^{\mathrm{pub}}$: $(n-k) \times n$ matrix
  such that $\mathrm{SHP} = \left(\mathrm{Id}_{n-k} \mid \mathrm{H}^{\mathrm{pub}}\right)$.
- Public Key: $(\mathrm{H}^{\mathrm{pub}}, t)$
- Private Key: $(\mathrm{S}, D_{\mathcal{G}}, \mathrm{P})$
  where $D_{\mathcal{G}}$ is an efficient *syndrome* decoding algorithm for $\mathcal{G}$.

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- Encryption: to encrypt message $\mathsf{e} \in \mathbb{F}_2^n$ of weight t, compute the syndrome

$$\mathsf{s} = \left(\mathsf{Id}_{n-k} \mid \mathsf{H}^{\mathsf{pub}}\right) \mathsf{e}^{\mathsf{T}}.$$

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- Decryption: compute

$$\mathsf{s}' = \mathsf{S}^{-1} s = \mathsf{HPe}^{\mathsf{T}},$$

use $D_{\mathcal{G}}$ to recover $\mathsf{e}' = \mathsf{Pe}^{\mathsf{T}}$, compute

$$\mathsf{e}^{\mathsf{T}} = \mathsf{P}^{-1}\mathsf{e}' = \mathsf{P}^{-1}\mathsf{Pe}^{\mathsf{T}}.$$

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Code-based Cryptography
McEliece and Niederreiter

Recommended parameters:

$$
\begin{aligned}
n &= 6960 \\
m &= 13 \\
t &= 119 \\
k &= n - mt = 5413
\end{aligned}
$$

Estimated security level: 266 bit.

Public key size: $(n-k)k$ bits $\approx 1,046,739$ bytes.

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Disadvantages of McEliece and Niederreiter:

- Large key size when using binary Goppa codes.

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Further improvements for code-based schemes:

Use codes with a more compact representation, e.g. cyclic codes.

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- Signature schemes, e.g., CFS: large (huge?) public keys.

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- Signature schemes, e.g., CFS: large (huge?) public keys.
- Cryptographic hash functions, e.g., FSB: no competitive performance.
- Pseudo random number generators: no competitive performance?

Multivariate Cryptography

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\begin{aligned}
5x_1^3x_2x_3^2 + 17x_2^4x_3 + 23x_1^2x_2^4 + 13x_1 + 12x_2 + 5 &= 0\\
12x_1^2x_2^3x_3 + 15x_1x_3^3 + 25x_2x_3^3 + 5x_1 + 6x_3 + 12 &= 0\\
28x_1x_2x_3^4 + 14x_2^3x_3^2 + 16x_1x_3 + 32x_2 + 7x_3 + 10 &= 0\\
54x_1^6x_3 + 2x_1^4 + 59x_1^2x_2^3 + 42x_1^2x_3^7 + x_1 + 17 &= 0
\end{aligned}
$$

Underlying problem:

Solving a system of m multivariate polynomial equations in n variables over $\mathbb{F}_q$ is called the **MP problem**.

Multivariate Cryptography
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Example

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Hardness:

The MP problem is an *NP-complete* problem even for multivariate *quadratic* systems and $q = 2$.

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Hardness:

The MP problem is an *NP-complete* problem even for multivariate *quadratic* systems and $q = 2$.

Notation:

For a set $f = (f_1, \ldots, f_m)$ of m quadratic polynomials in n variables over $\mathbb{F}_2$, let $f(x) = (f_1(x), \ldots, f_m(x)) \in \mathbb{F}_2^m$ be the solution vector of the evaluation of f for $x \in \mathbb{F}_2^n$.

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Definition ($\mathcal{MQ}$ over $\mathbb{F}_2$)

Let $\mathcal{MQ}(\mathbb{F}_2^n, \mathbb{F}_2^m)$ be the set of all systems of quadratic equations in n variables and m equations over $\mathbb{F}_2$.
We call one element $P \in \mathcal{MQ}(\mathbb{F}_2^n, \mathbb{F}_2^m)$ an instance of $\mathcal{MQ}$ over $\mathbb{F}_2$.

Multivariate Cryptography
Basic Idea for Multivariate Public Key Cryptography (MPKC)

- System Parameters: $m, n, \in \mathbb{N}$.
- Key Generation: choose "random" $f \in \mathcal{MQ}(\mathbb{F}_2^n, \mathbb{F}_2^m)$ such that f^{-1} is secretly known.
- Public Key: f.
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Problem:

How do you find f and f^{-1} such that f is a hard instance of $\mathcal{MQ}$?

Design pattern

Usually, f is constructed as a sequence of invertible functions, e.g.,

$$f = r \circ s \circ t$$

with r and t multivariate linear and
s quadratic with a easy-to-invert structure.

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Recent secure (i.e., not yet broken?) **examples:**

- Rainbow signature scheme,
- Quartz or HFEv- signature scheme,
- PMI+ public key encryption scheme.

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$\left.\begin{array}{l} \text{Rainbow} \\ \text{Quartz / HFEv-} \end{array}\right\}$ Easier to construct.

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Multivariate Public Key Cryptography (MPKC)

Further MQ schemes:

- symmetric encryption schemes,
- cryptographic hash functions, and
- pseudo random number generators.

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Concerns about MQ schemes:

- Most public-key encryption schemes have been broken!
- Efficient (sparse) $\mathcal{MQ}$ instances have problems with randomness!

Hash-based Cryptography

Hash-based Cryptography
Introduction

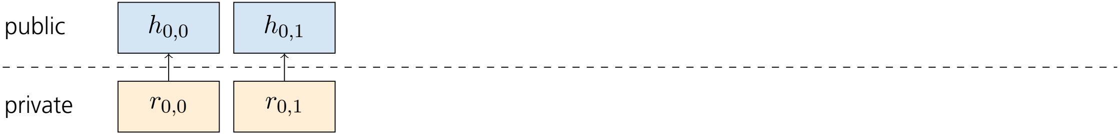
Basic idea:

Computing pre-images of a cryptographic hash function remains hard also for quantum computers (Grover).

$\Rightarrow$ Use pre-image as private key, hash-value as public key.

Hash-based Cryptography

Lamport and Merkle



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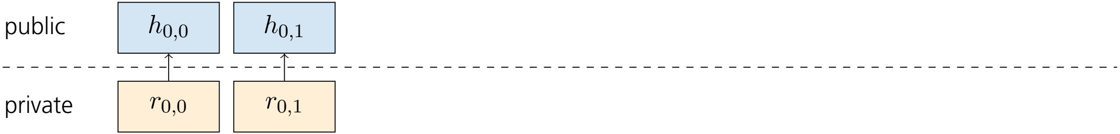

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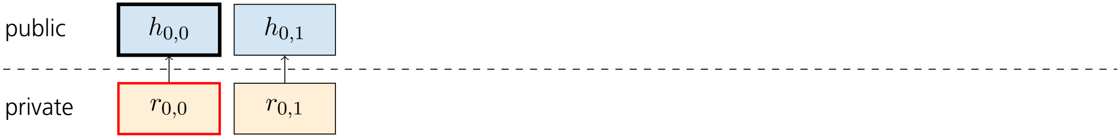

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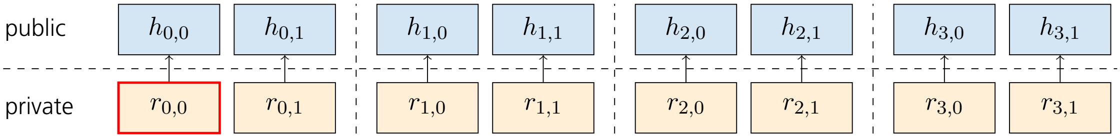

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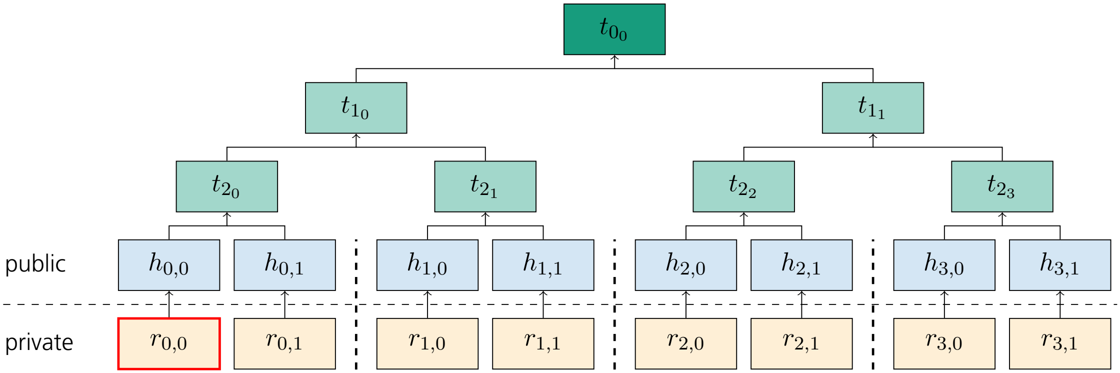

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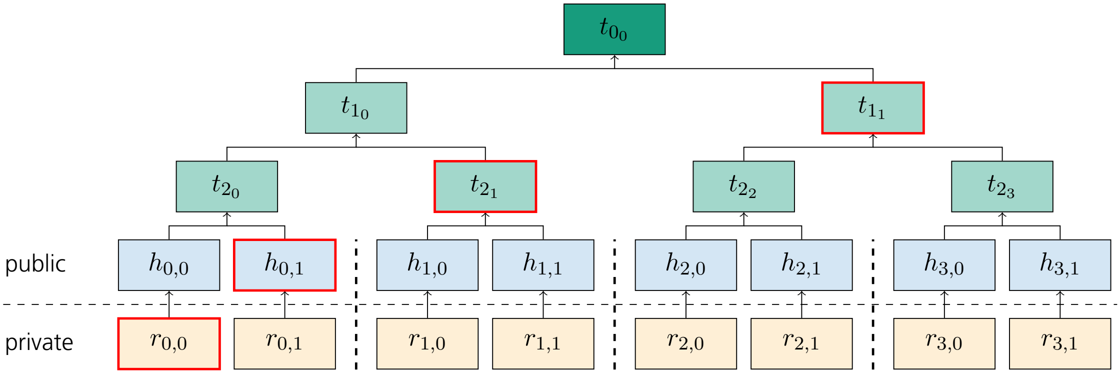

Message: 0_b

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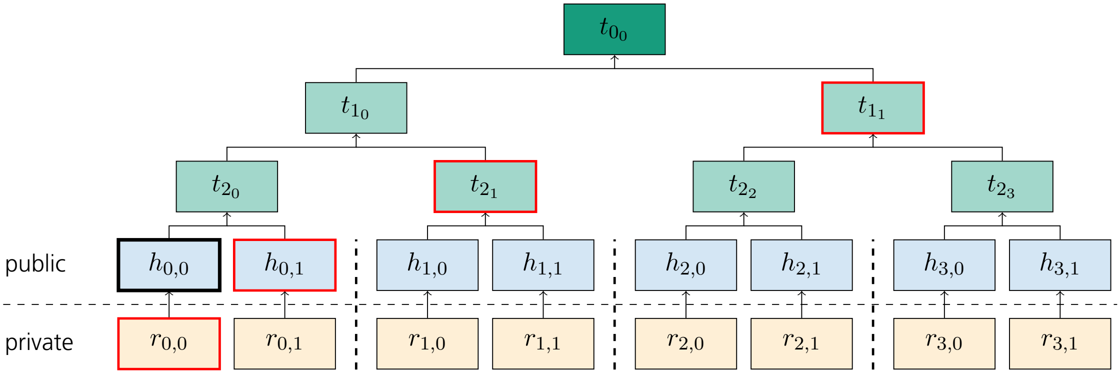

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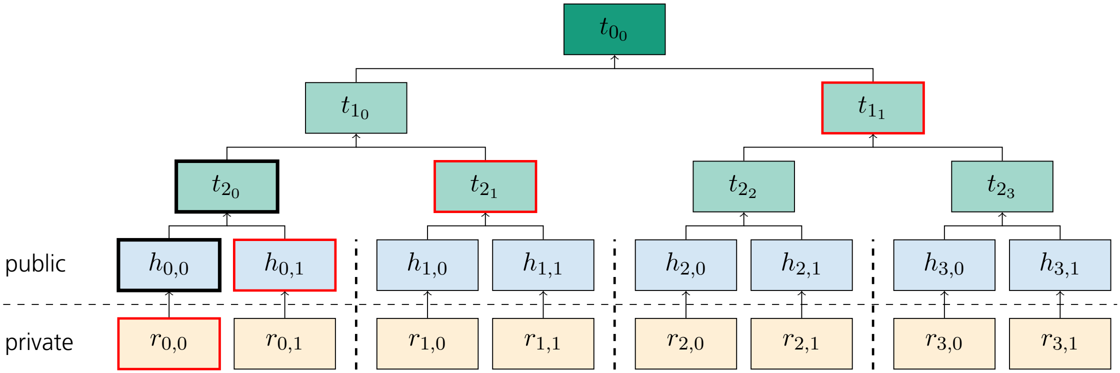

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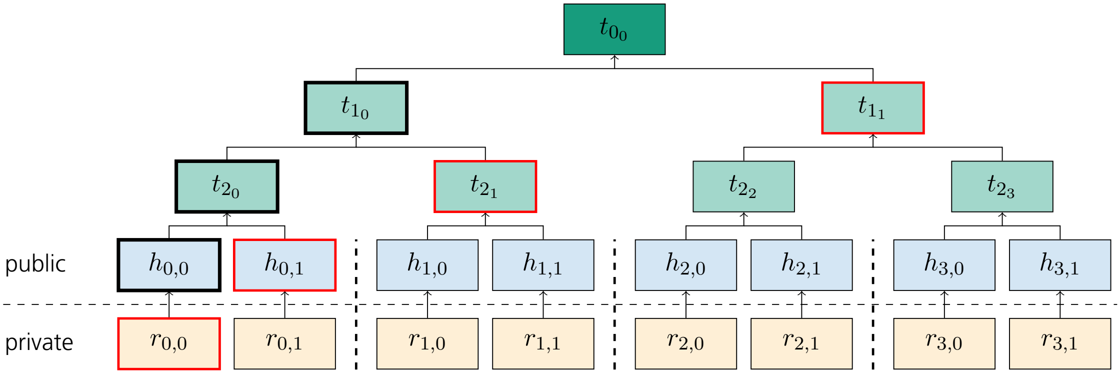

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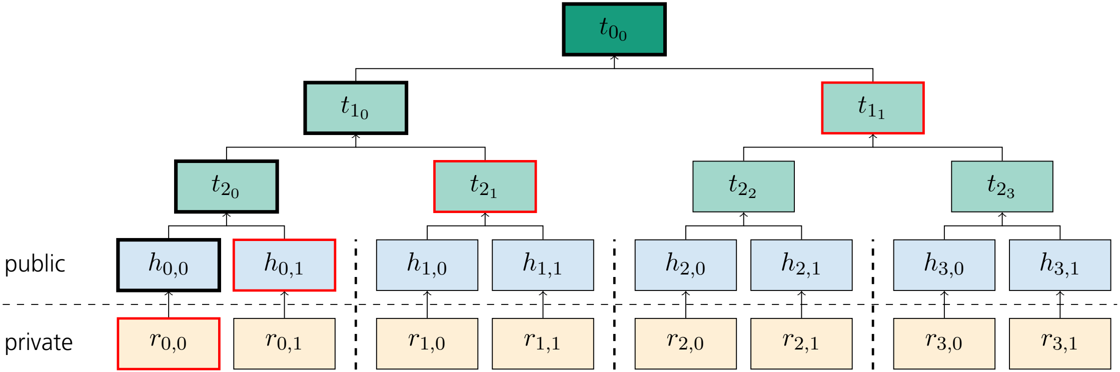

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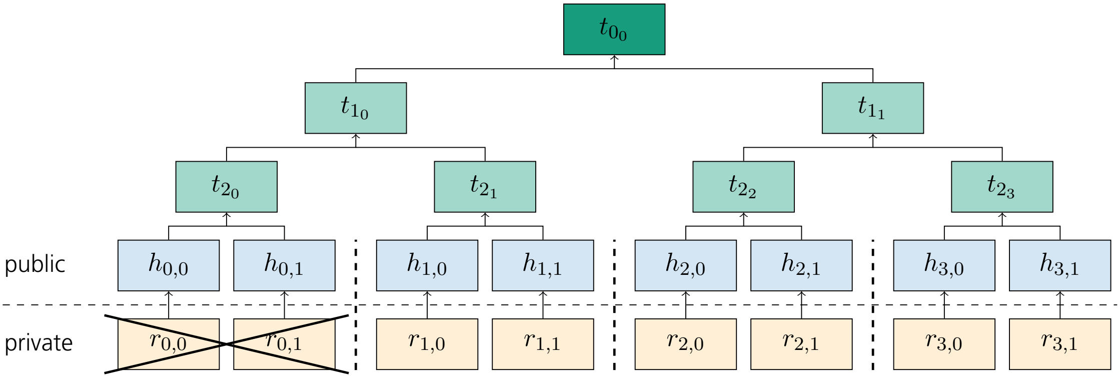

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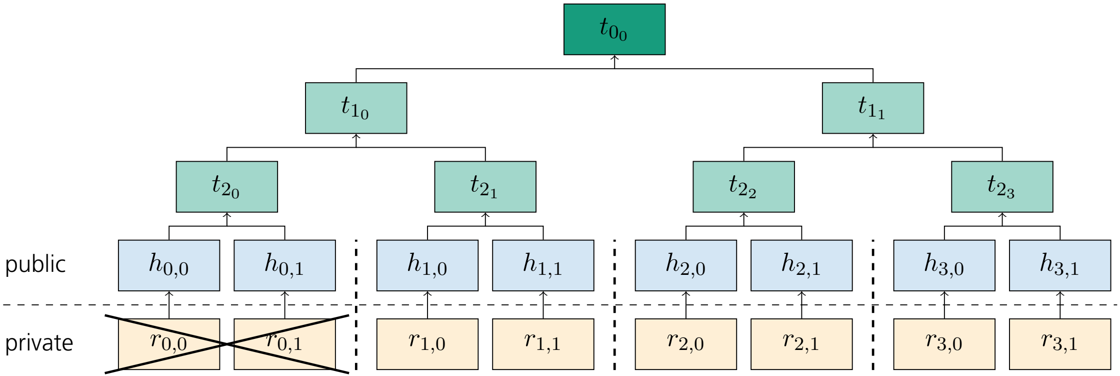

Message: 1_b

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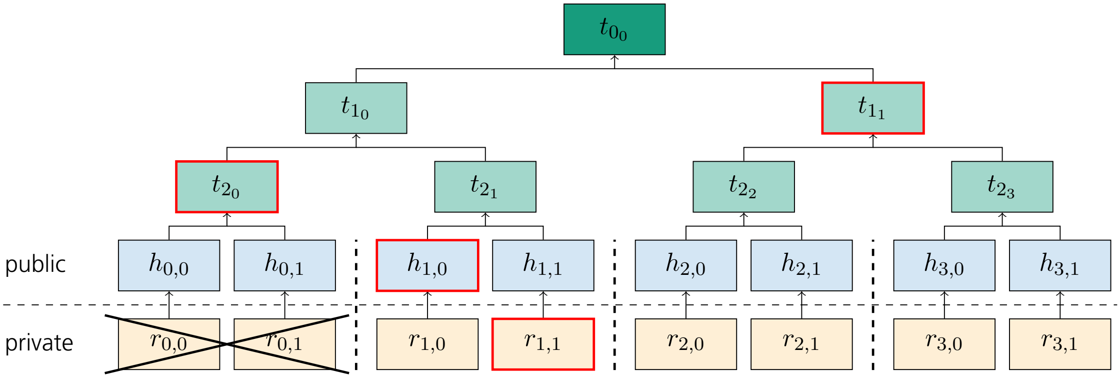

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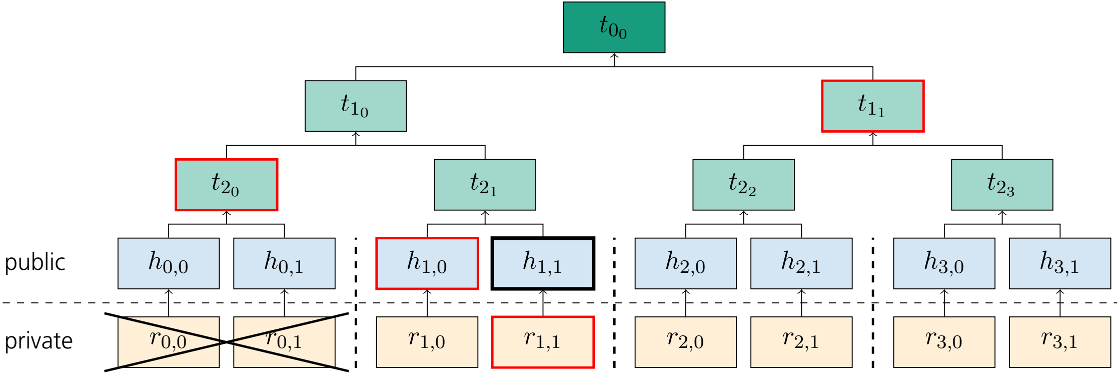

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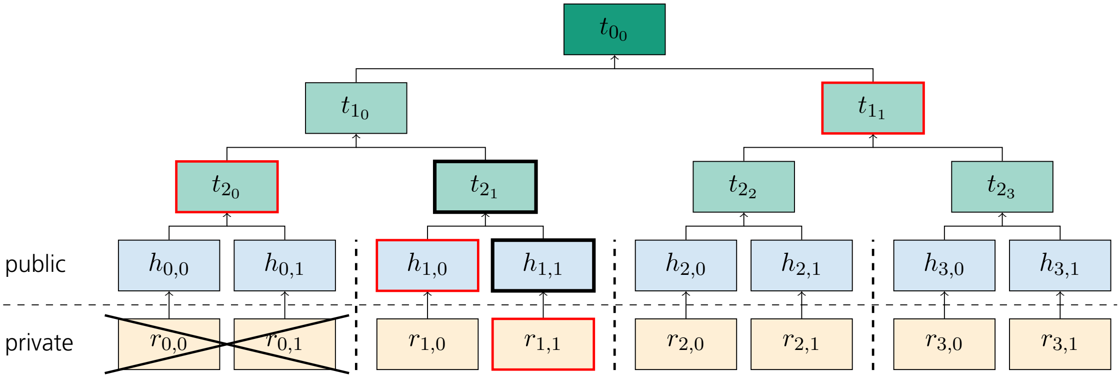

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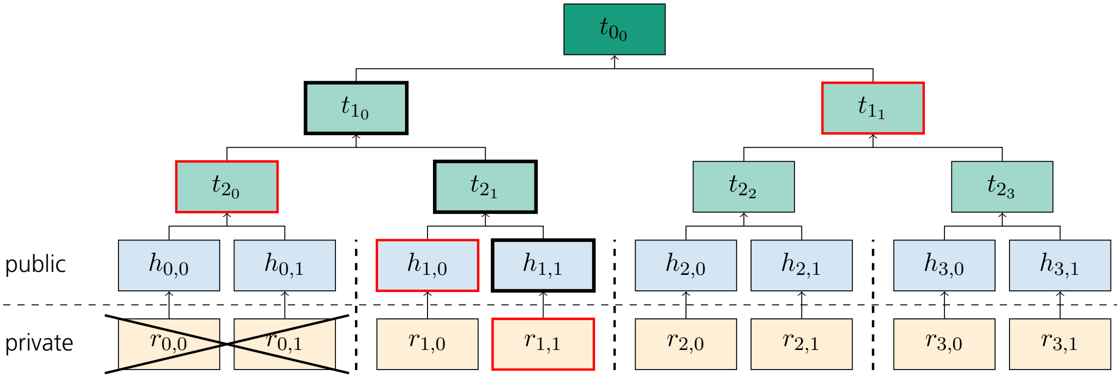

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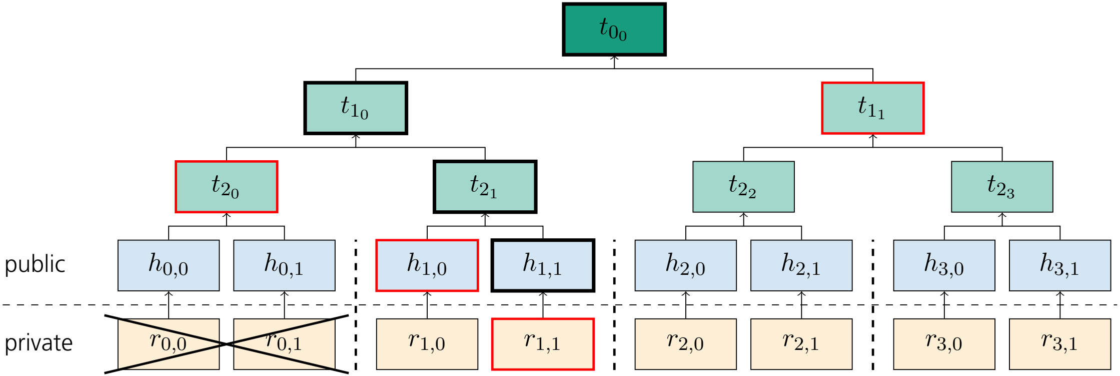

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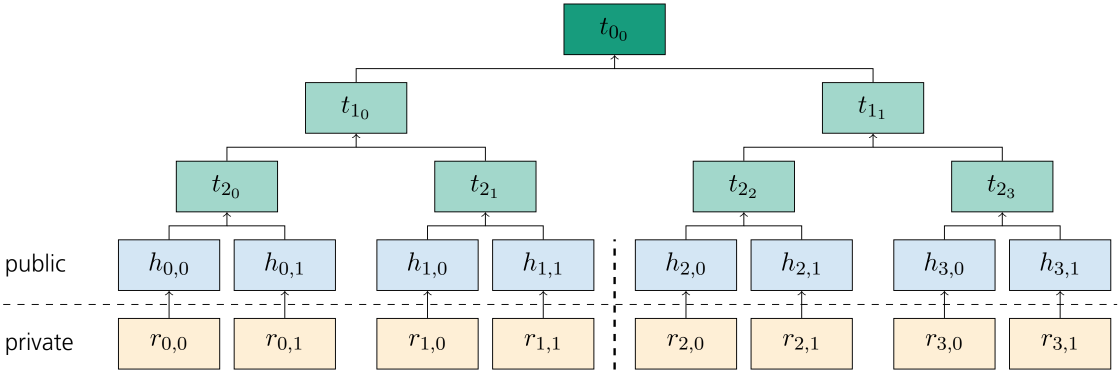

Message: 10_b

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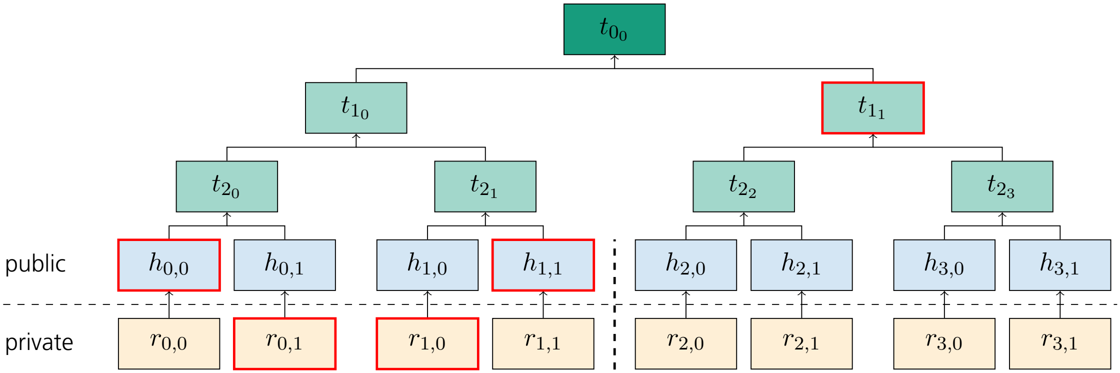

Message: 10_b

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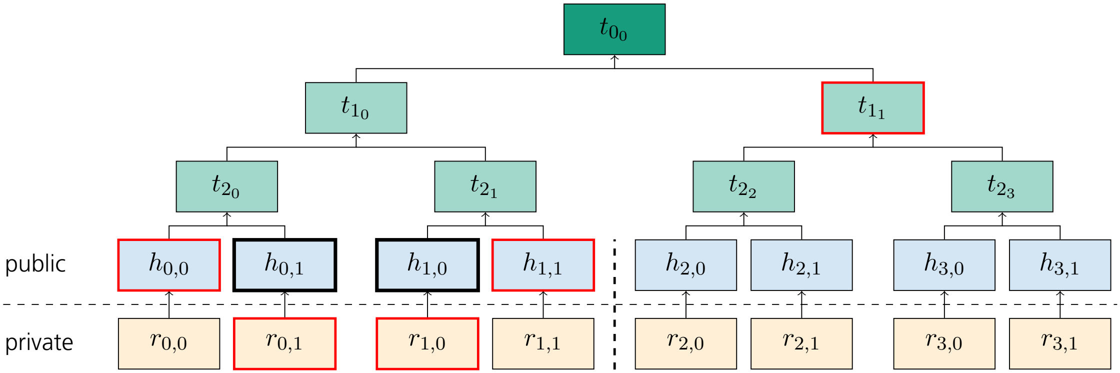

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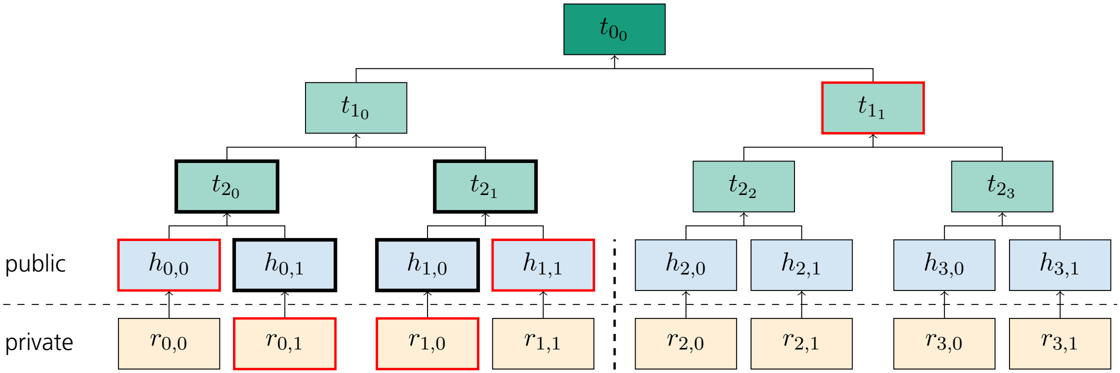

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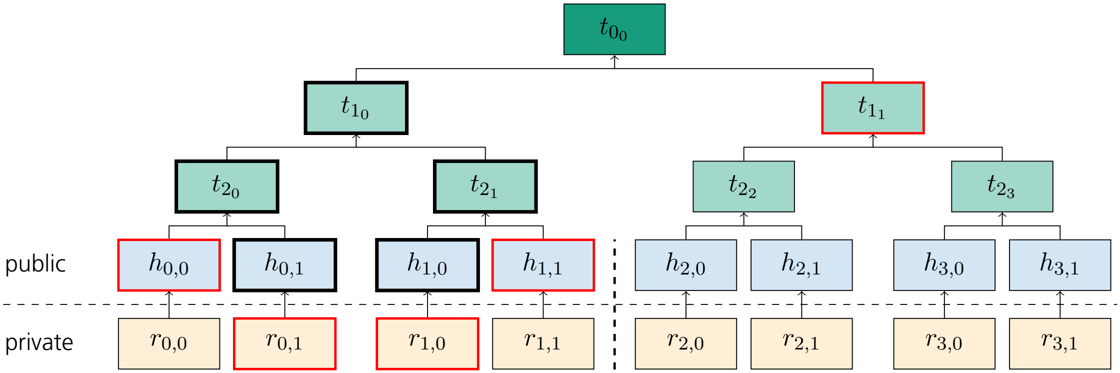

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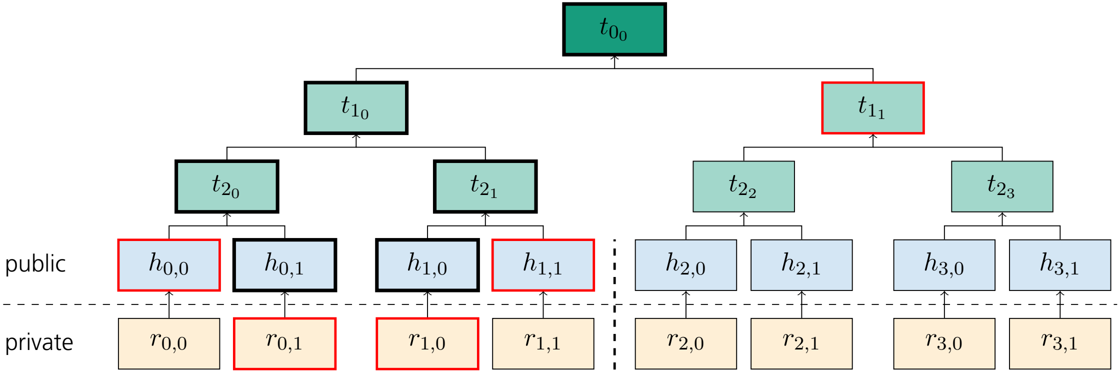

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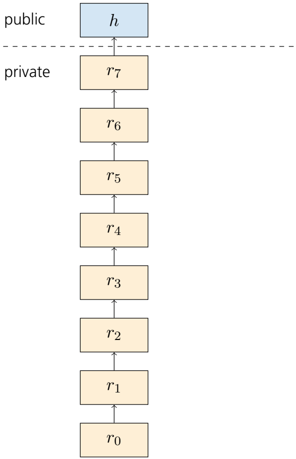

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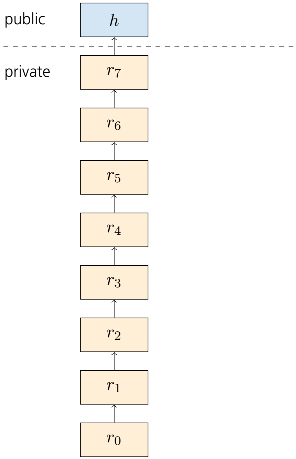
Message: $101_b = 5$



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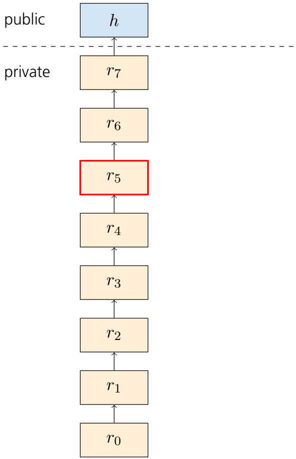

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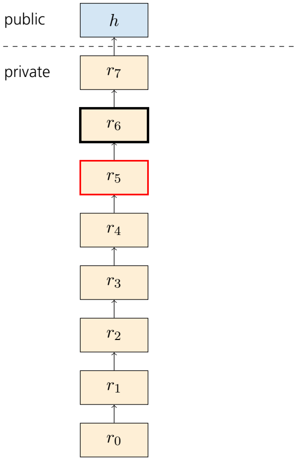

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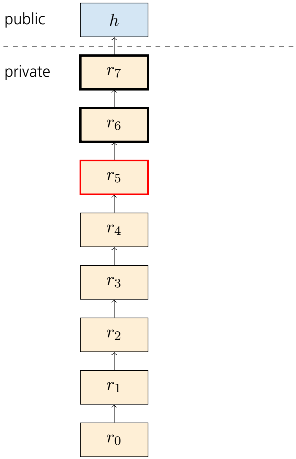

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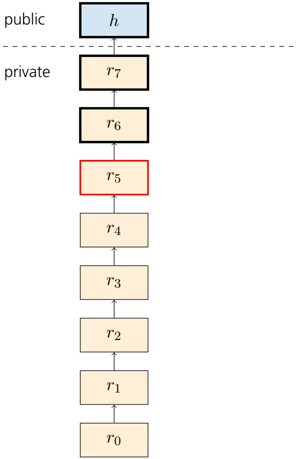

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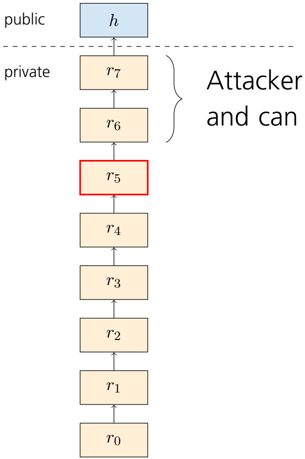

Attacker learns private keys and can sign 110_b and 111_b!

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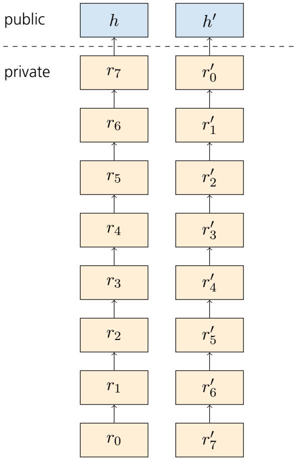

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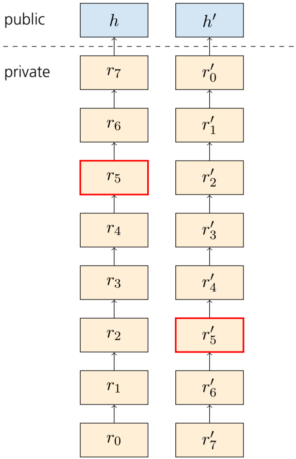

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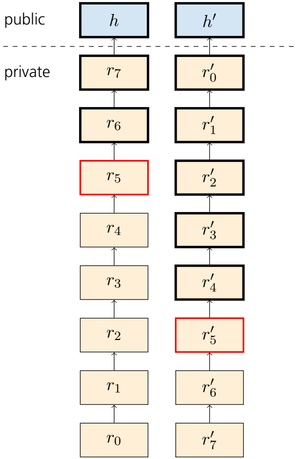
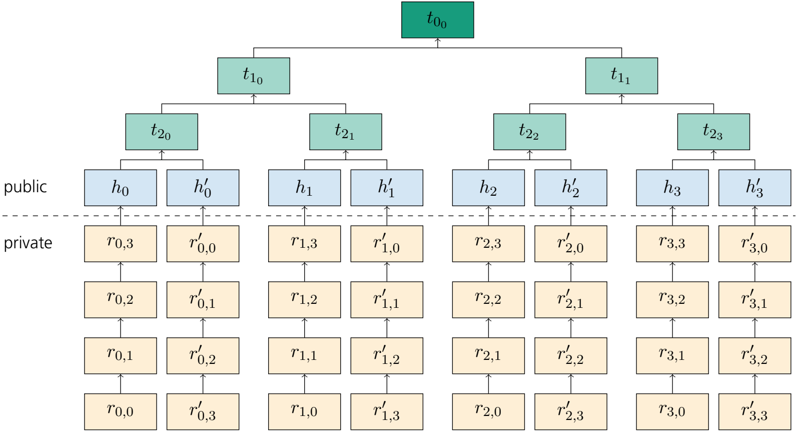


Hash-based Cryptography

(Simplified) Winternitz and Merkle Tree



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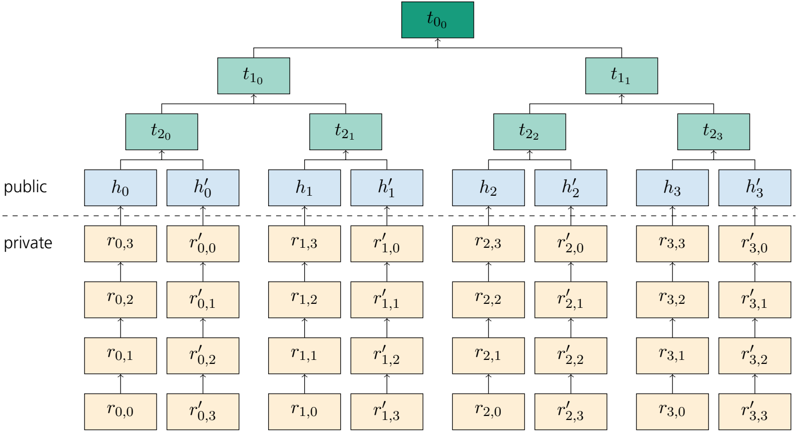

Message: $01_b = 1$

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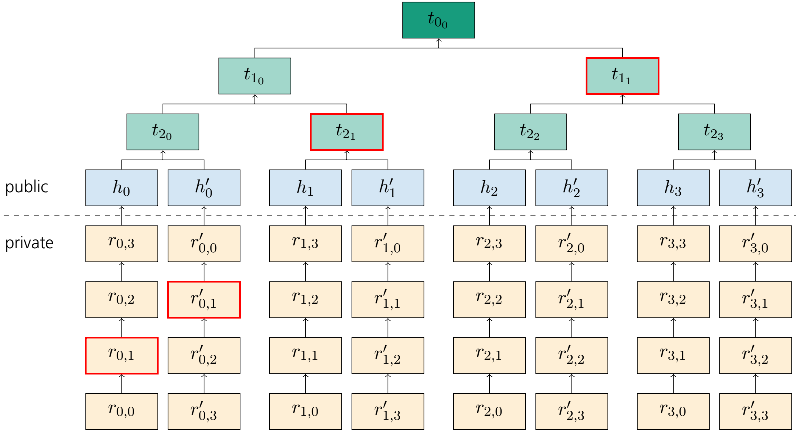

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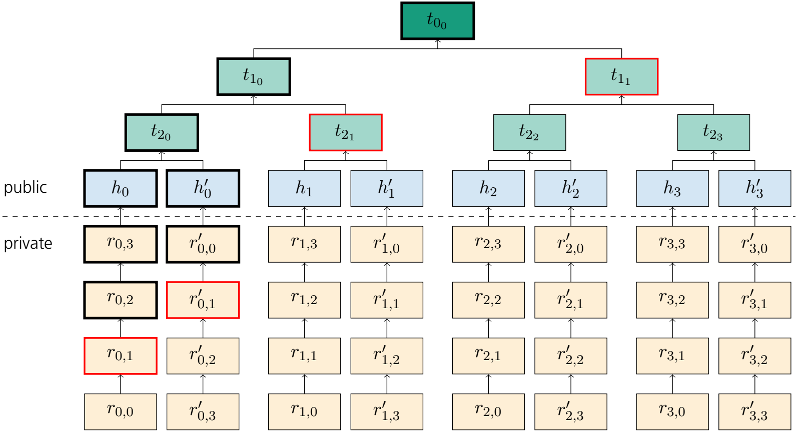

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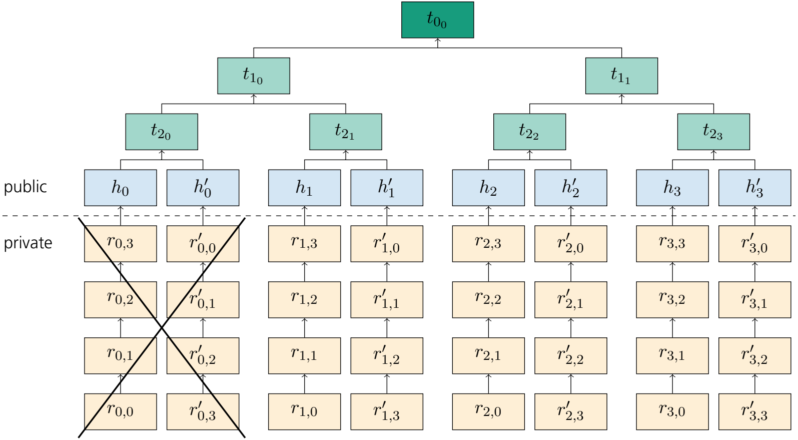

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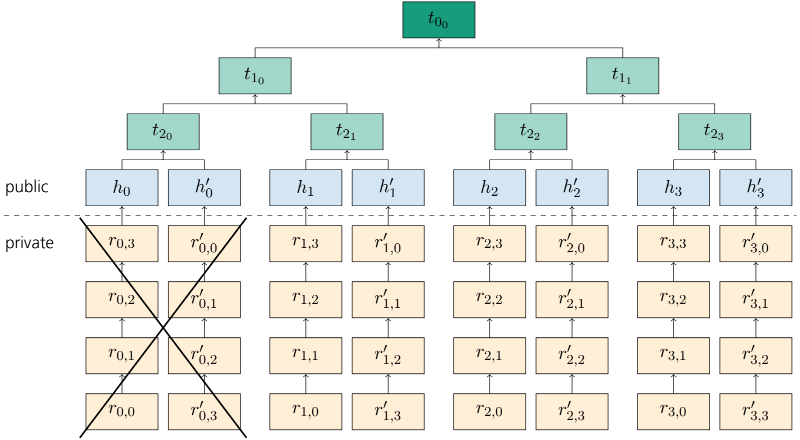

Message: $11_b = 3$

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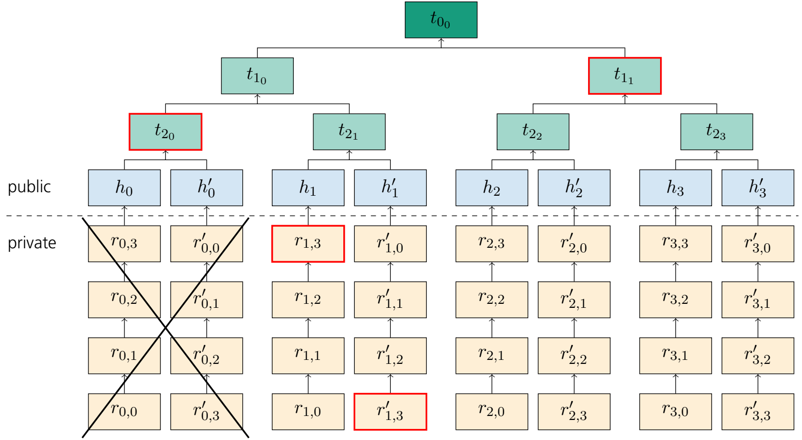

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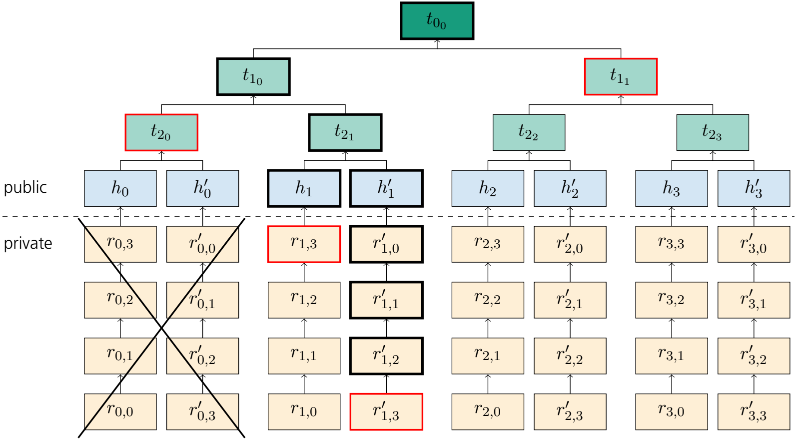

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Hash-based Cryptography

Summary:

- Only helpful for Signatures.

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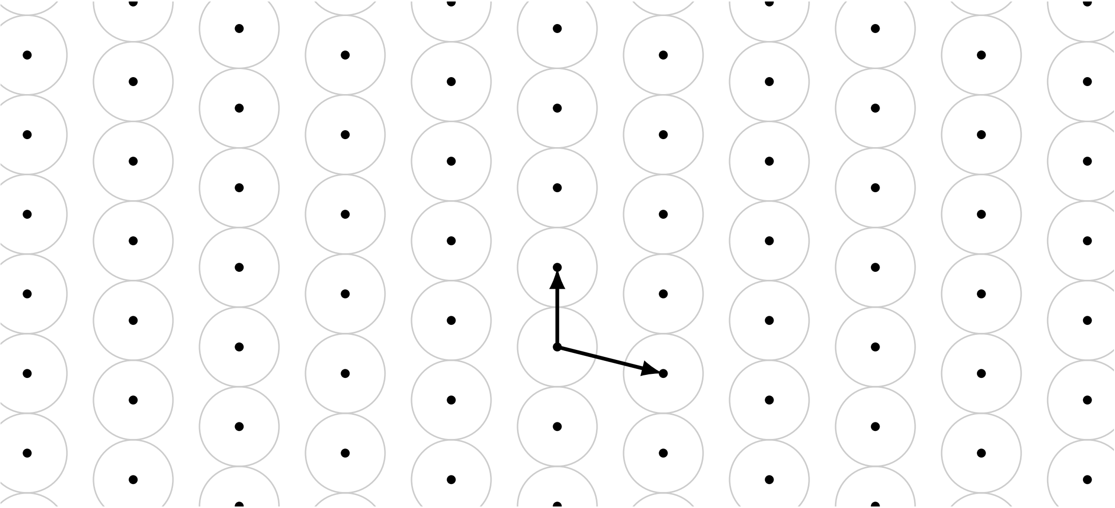
Lattice-based Cryptography

Lattice-based Cryptography
Introduction



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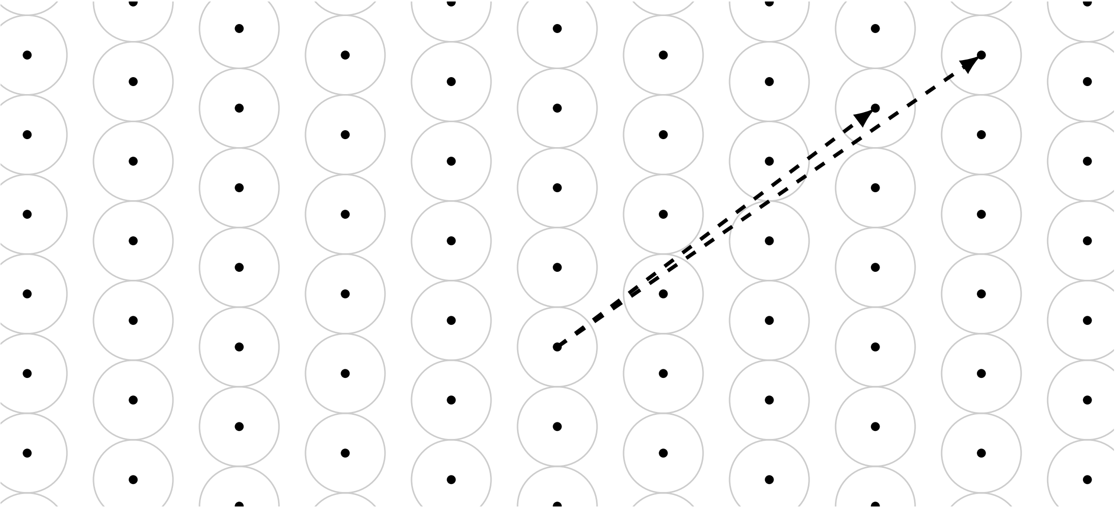

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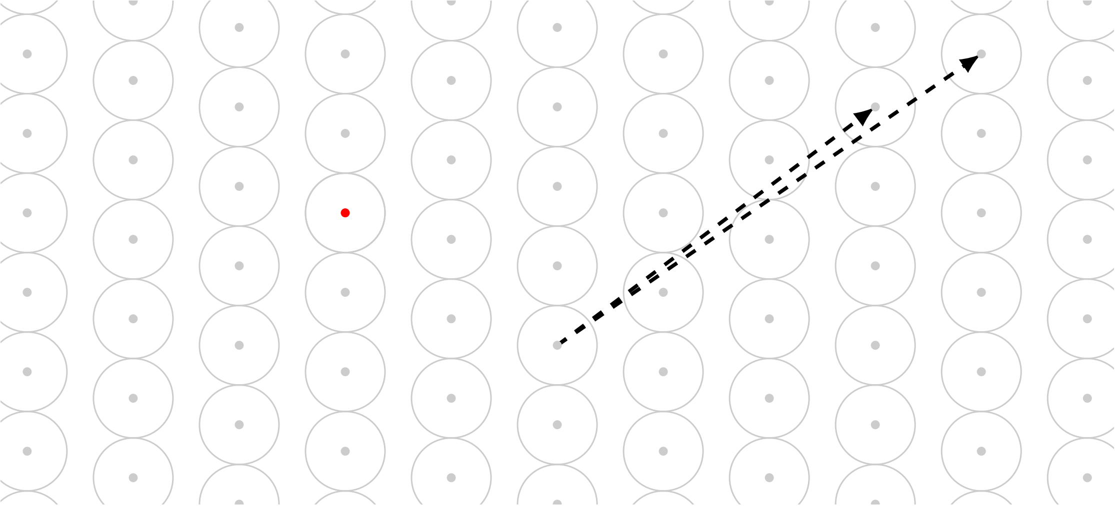

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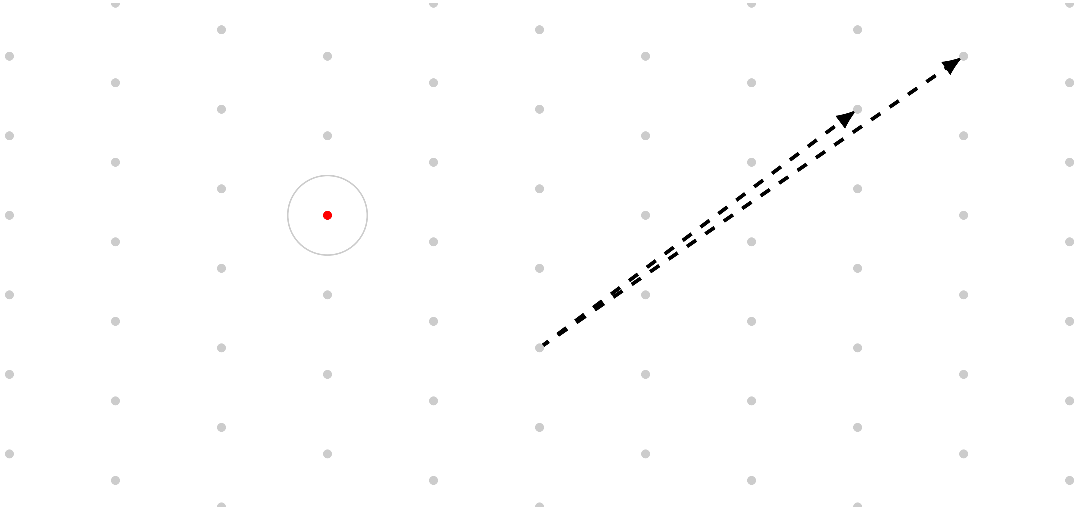

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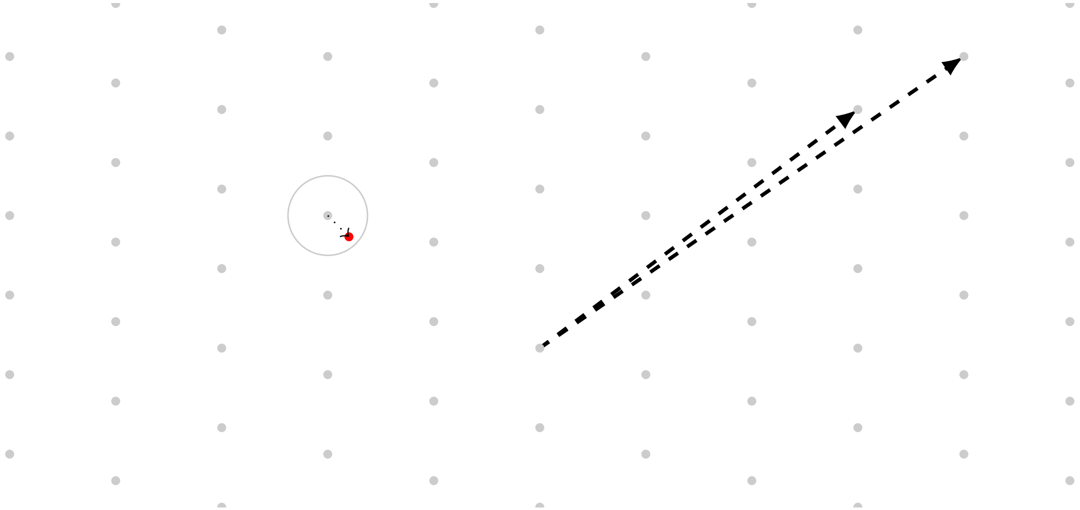

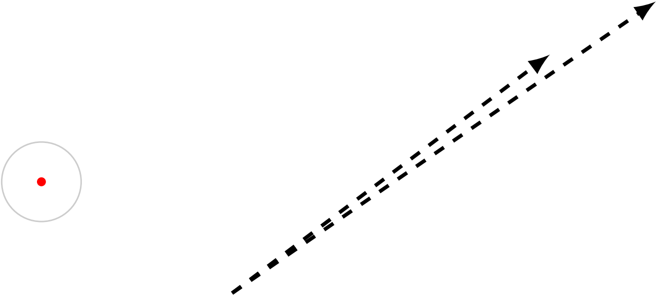

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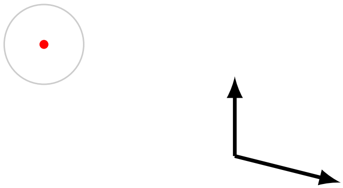

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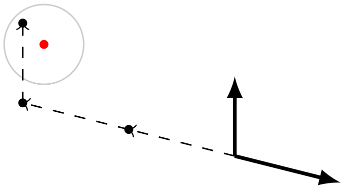

Fraunhofer SIT

Underlying hard problems:

- CVP: closest vector problem,
- SVP: shortest vector problem,
- LWE: learning with errors.

Lattice-based Cryptography
Introduction

Underlying hard problems:

- CVP: closest vector problem,
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Popular lattice-based schemes:

- public key encryption: NTRU, NTRU prime;
- key exchange: New Hope (experimentally used by Google).

Security proofs of lattice-based schemes:

- There are security proofs and
  worst-case to average-case reductions.

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- There are security proofs and worst-case to average-case reductions.
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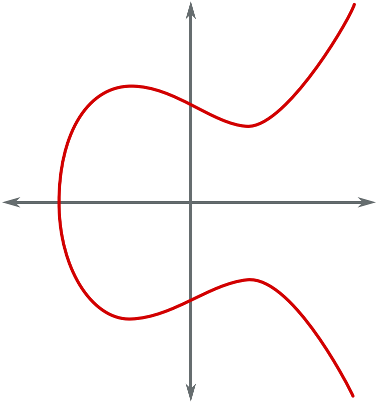
Problems with lattice-based schemes:

- Attack-complexity not yet deeply understood,
- attacks are improved frequently.

Supersingular Isogenies



$$y^2 = x^3 - x + 1$$

Supersingular Isogenies

Overview

$$\begin{tikzcd} & E_A \arrow[dr, "\psi'(E_A)"] & \\ E \arrow[ur, "\phi(E)"] \arrow[dr, "\psi(E)"'] & & E_{AB} \\ & E_B \arrow[ur, "\phi'(E_B)"'] & \end{tikzcd}$$

Basic idea:

- Use secret mappings (isogenies) between elliptic curves to compute a shared secret.
- Does not operate on points of a curve but on curves using maps.

Supersingular Isogenies
Overview

$$\begin{tikzcd}
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E \arrow[ur, "\phi(E)"] \arrow[dr, "\psi(E)"'] & & E_{AB} \\
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\end{tikzcd}$$

Features:

- **DH-like PQ key exchange scheme.**
- \+ Small communication overhead.
- – High computational cost.

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Supersingular Isogenies
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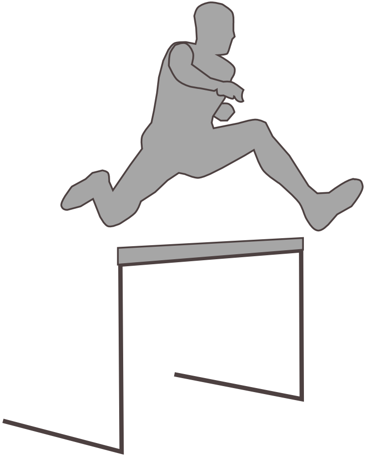
$$

Problems:

- Very recent proposal; security not yet well understood.
- First proposal with *ordinary* curves broken by quantum computers.
- New proposal using *supersingular* curves under examination.

Performance and Challenges



Performance and Challenges
Recommendations

Initial recommendations from the “PQCRYPTO project” (2015) [1]:

- Symmetric Encryption:
  - AES-256,
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  - McEliece with binary Goppa codes using length $n = 6960$, dimension $k = 5413$, and adding $t = 119$ errors.
- Public-key Signatures:
  - XMSS (with state),
  - SPHINCS-256 (stateless).

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Performance and Challenges

NIST Post-Quantum Cryptography Standardization

Time line:

Feb. 2016	Announcement at PQCrypto 2016

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Performance and Challenges
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2 years later	Draft Standards ready

Performance and Challenges

NIST Post-Quantum Cryptography Standardization

Round 1 Submissions:

Family	Signatures	KEM/Encryption	sum
lattice-based	5	23	28
code-based	3	17	20
multivariate	7	3	10
hash-based	2		2
"others"	3	6	9
sum	20	49	69

Performance and Challenges
Cost of PQ Schemes

Scheme	Public key size (bytes)	Data size (bytes)
Classical schemes:		
• RSA:		
– RSA-2048	256	256
– RSA-4096	512	512
• ECC:		
– 256-bit	32	32
– 512-bit	64	64
• Key exchange:		
– DH	—	256 – 512
– ECDH	—	32 – 64

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Performance and Challenges
Cost of PQ Schemes

Scheme	Public key size (bytes)	Data size (bytes)
Public-key signatures:		
• Hash based:		
– XMSS (stateful)	64	2,500 – 2,820
– SPHINCS (state free)	1,056	41,000
• Multivariate based:		
– HFEv-	500,000 – 1,000,000	25 – 32
– Rainbow	148,500 – 1,321,000	64 – 147
• Lattice based:		
– Dilithium	896 – 1760	1386 – 3365
– qTESLA	2,976 – 6,432	2,720 – 5,920

Performance and Challenges
Cost of PQ Schemes

Scheme	Public key size (bytes)	Data size (bytes)
Public-key encryption:		
• Code based:		
- McEliece (binary Goppa codes)	958,482 – 1,046,739	187 – 194
- McEliece (QC-MDPC codes)	4,097	8,226
• Lattice based:		
– NTRUEncrypt	1,023 – 4,097	1023 – 4,097
– Kyber (KEM)	1,088	1,184

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Performance and Challenges

Cost of PQ Schemes

Scheme	Public key size (bytes)	Data size (bytes)
Key exchange:		
• Lattice based:		
– NewHope	—	1,824 – 2,048
– Kyber (KEX)	—	1,184 – 2,368
• Supersingular isogenies:		
– SIDH	—	564

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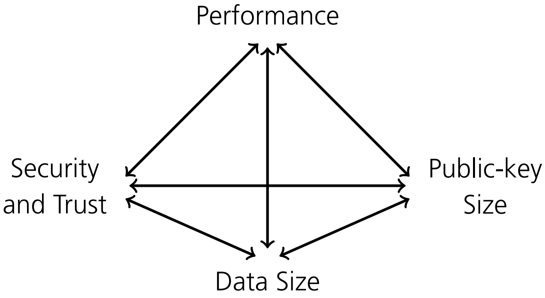
Performance and Challenges
Relative Performance

Family	Key Generation	Public Key Encryption/Verification	Private Key Decryption/Signing
Code based:	slow	fast	medium
Multivariate:	slow	fast	medium
Hash based:	slow	fast	slow
Lattice based:	fast	fast	fast
Isogenies:	————	slow (key exchange)	————
ECC-256	fast	medium	fast
RSA-3072	slow	fast	slow

Performance and Challenges
Challenges

Open research questions:

- Make trusted schemes more efficient.
- Make efficient schemes more reliable.



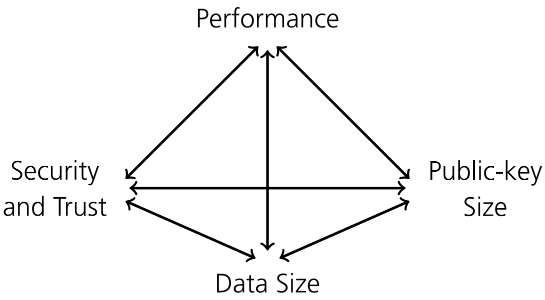
Performance and Challenges
Challenges

Open research questions:

- Make trusted schemes more efficient.
- Make efficient schemes more reliable.

Real-world PQC:

- Investigate the usability of PQC schemes in real-world applications.
- Prepare applications for the transition to PQC. $\Rightarrow$ crypto-agility



Thank you for your attention!

Literature

D. Augot, L. Batina, D. J. Bernstein, J. Bos, J. Buchmann, W. Castryck, O. Dunkelman, T. Güneysu, S. Gueron, A. Hülsing, T. Lange, M. S. E. Mohamed, C. Rechberger, P. Schwabe, N. Sendrier, F. Vercauteren, and B.-Y. Yang. *Initial recommendations of long-term secure post-quantum systems*. Tech. rep. `http://pqcrypto.eu.org/docs/initial-recommendations.pdf`. PQCRYPTO Horizon 2020 ICT-645622, Sept. 2015.

D. J. Bernstein, J. Buchmann, and E. Dahmen, eds. *Post Quantum Cryptography*. Springer, 2008.

D. Deutsch. "Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer". In: *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* 400.1818 (1985), pp. 97–117.

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Contact Information

Dr. Ruben Niederhagen

Cyber-Physical System Security

Fraunhofer-Institute for
Secure Information Technology

Address: Rheinstraße 75
64295 Darmstadt
Germany
Internet: http://www.sit.fraunhofer.de

Phone: +49 6151 869-135
Fax: +49 6151 869-224
E-Mail: ruben.niederhagen@sit.fraunhofer.de